Single Degree of Freedom

All we have to do for these problems is find E.O.M, solve for ω_n

$$M\ddot{z} + K\dot{z} = 0 \qquad \omega_n = \sqrt{\frac{K}{M}}$$

Remember to convert θ into x using r or appropriate.

Free Vibration

No input force or displacement

Use formula sheet, pick equation depending on ζ (should be light damping)

Apply initial conditions to solve for $z(t)$ or $\dot{z}(t)$.

Harmonic Response

Get E.O.M, sub in $Ae^{i\omega t}$ e.g. $Fe^{i\omega t}$ or $Ze^{i\omega t}$

solve for z^*, $H(\omega) = \frac{|Z^*|}{|F|}$ or $\frac{|Z^*|}{|R|}$

For most problems, equations in formula sheet

Or use to check!

N-Degree of Freedom

Get E.O.M's for each mass or J.

Put E.O.M's into matrix form

$$\cancel{[M] + [x} \; [M]\{\ddot{z}\} + [k]\{z\} = 0$$

Form [Z] matrix:

$$[Z] = [k] - [M]\omega^2$$

For ω_n's

Solve quadratic from $\det[Z] = 0$ for ω_n^2

For Mode shape:

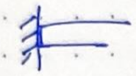
Sub ω_n back into [Z] matrix, set either Z_1 or Z_2 as 1

Beam Vibration

Hopefully all we have to do is create $[Z]\{C\}=0$ matrix and draw mode shapes

Use formula sheet for $Y(x)$, $\frac{dY}{dx}$, $\frac{d^2Y}{dx^2}$, $\frac{d^3Y}{dx^3}$

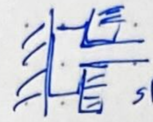
Solve for B.C's and put in matrix for, c_1, c_2, c_3, c_4



$Y=0$
$\frac{dy}{dx}=0$

$Y=0$
$\frac{d^2y}{dx^2}=0$

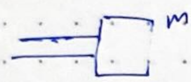
$\frac{d^2y}{dx^2}=0$
$\frac{d^3y}{dx^3}=0$



slider

$\frac{dy}{dx}=0$ $\frac{d^3y}{dx^3}=0$

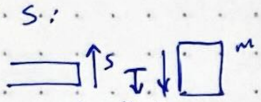
IF UNLUCKY:



$x=L$

$S\neq0$ $M\neq0$

For S:



$x=L$

$S=m\left.\frac{d^2y}{dt^2}\right|_{x=L}$

In formula sheet

$S=EI\left.\frac{d^3y}{dx^3}\right|_{x=L}$

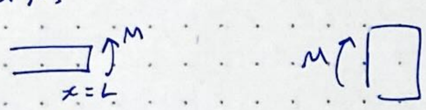
In formula sheet

Sub in, $y(x,t)=Y(x)\cos\omega t$, also $\omega^2=\frac{\lambda^4 EI}{\rho A}$

and equate.

$$\boxed{\left.\frac{d^3Y}{dx^3}\right|_{x=L}+\frac{m(\lambda L)^4}{\rho AL^4}Y(L)=0}$$

For M:



$x=L$

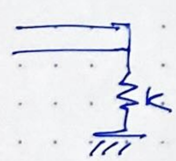
$M=I_M\frac{\partial^2\theta}{\partial t}$, $\theta(t)=\frac{\partial y}{\partial x}$
$=\frac{dY}{dx}\cos\omega t$

$M=-EI\left.\frac{\partial^2y}{\partial x^2}\right|_{x=L}$

so

$\frac{\partial^2\theta}{\partial t^2}=-\omega^2\frac{dY}{dx}\cos\omega t$

$$\boxed{\left.\frac{d^2Y}{dx^3}\right|_{x=L}-\frac{M(\lambda L)^4}{\rho AL^4}\left(\frac{dY}{dx}\right)_{x=L}=0}$$



$M=0$
$S\neq0$

$S=EI\frac{d^3y}{dx^3}$

$y(x)$ $\quad$ $kY(x)$

$-S-kY(0)=0$

$$\boxed{kY(x)+EI\frac{d^3Y(x)}{dx^3}=0}$$

$S=0$
$M\neq0$

Vibration Isolation

Force: $T_f = \left|\frac{Q^*}{P}\right|$ Displacement: $T_D = \left|\frac{X^*}{Y}\right|$ Both share same expression
IN FORMULA SHEET

for T_{max} at ω_{min}, maximum isolator stiffness $\Rightarrow$ $K = m\omega_n^2 = \frac{m T_{max}\,\omega_{min}^2}{1 + T_{max}}$

There may also be a maximum static deflection $X_0 = \frac{mg}{K}$

Design procedure: Find CoM, find isolator positions, estimate load on each isolator,
for each isolator: calculate max K, check doesnt exceed X_0, repeat.

Isolation efficiency = $(1-T) \times 100\%$

Approximate Methods:

Rayleighs Method:

always greater than real values for ω_n

$$U_{max} = T_{max}$$

Sum of $\frac{1}{2} K_n Z_n^2$

Sum of $\frac{1}{2}\omega^2 (M_n Z_n^2)$

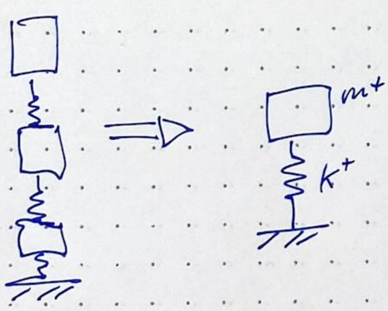
Equate and use mode shape guess to obtain ω_n

~~M~~ore mass spring systems either use $Z_3 > Z_2 > Z_1$ etc. (in phase)
or guess static deflection

For beams, use U_{max} and T_{max} in formula sheet, if there are masses on the beam $T_{max} = \boxed{\int m} + \cancel{\tfrac{1}{2}}$ sum of mass energies $\frac{1}{2}\omega^2 m z^2$

Guess Mode shape as: Cx^2 ~~or~~ for cantilever, $\sin\frac{\pi x}{L}$ for simple supports.

SDOF Approx:



By redrawing the system as 1 D.O.F
equate $U_{max} = U_{max}^+$ and $T_{max} = T_{max}^+$
to obtain m^+ and k^+

~~Use~~ Use same mode shape guesses

m^+ and k^+ can be used to find ω_n