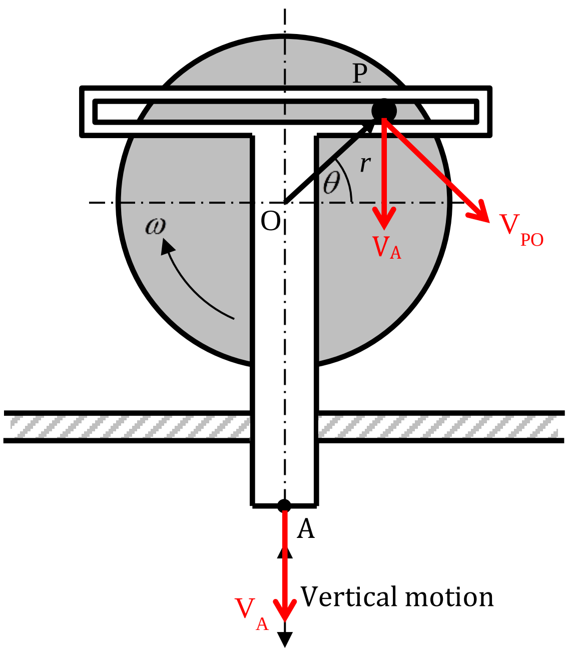
Department of Mechanical, Materials and Manufacturing Engineering

University of Nottingham

**MACHINE DYNAMICS**

## SHEET 3: KINEMATICS OF LINKAGE MECHANISMS - SOLUTION

1) Velocity Analysis




Velocity components shown

Velocity of A is calculated by noticing that it is the same as the vertical component of the velocity of P relative to fixed point O. i.e.

$$V_A = V_{PO}\cos 30^o \qquad (1)$$

where $V_{PO}$ is the tangential of P having magnitude

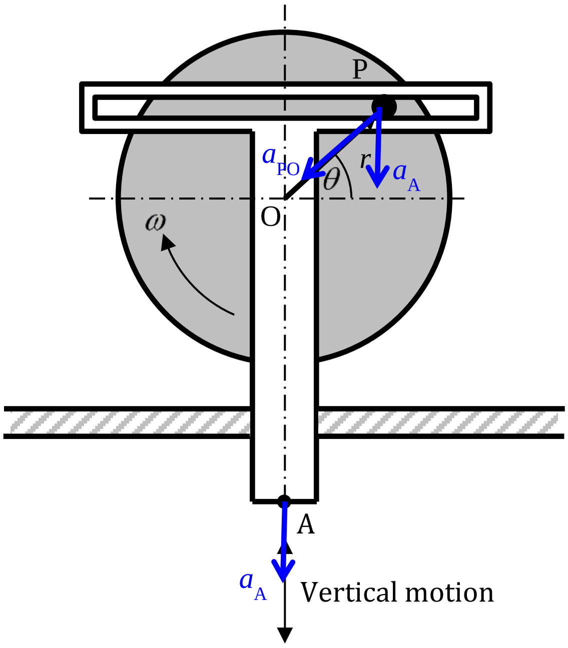
$$V_{PO} = r\omega = 0.12 \text{ m/s}$$

Using equation (1) gives

$$V_A = r\omega\cos 30^o = 0.104 \text{ m/s}$$

The direction of the velocity is downwards.

Acceleration Analysis




Acceleration components shown

Acceleration of A is calculated by noticing that it is the same as the vertical component of the acceleration of P relative to fixed point O. i.e.

$$a_A = a_{PO}\sin 30^o \qquad (2)$$

where $a_{PO}$ is the centripetal acceleration acting towards O with magnitude

$$a_{PO} = r\omega^2 = 0.72 \text{ m}/s^2$$

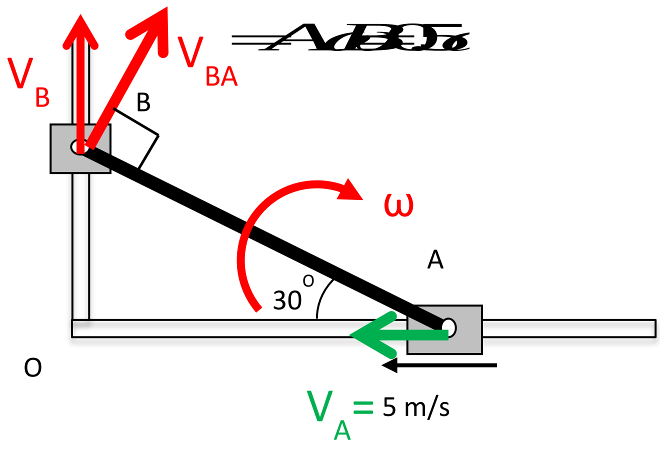
Using equation (2) gives

$$a_A = r\omega^2 \sin 30^o = 0.36 \text{ m}/s^2$$

The direction of the acceleration is downwards.

2)

Velocity Analysis




Velocity components shown

Velocity of B calculated using vector equation:

$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA} \qquad (1)$$

Resolve Eq.(1) in horizontal direction

$$\xrightarrow{+} \Sigma X: \quad 0 = -5 + V_{\mathrm{BA}} \sin 30^{o}$$

so

$$V_{\mathrm{B}A} = 5/\sin 30^{o} = 10\mathrm{m/s}$$

Also

$$\omega = \frac{V_{\mathrm{B}A}}{AB} = 10/0.5 = 20\mathrm{rad/s}$$

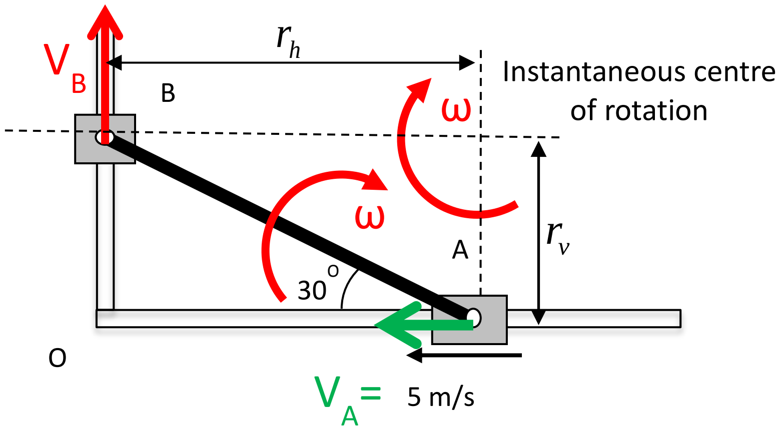
Resolve Eq. (2) in vertical direction

$$\uparrow^{+} \Sigma Y: \quad V_{\mathrm{B}} = 0 + V_{\mathrm{BA}} \cos 30^{o}$$

So

$$V_{\mathrm{B}} = 10 \cos 30^{o} = 8.66\mathrm{m/s}$$

The velocity of B can also be calculated from the instantaneous centre of rotation for link AB. At the instant shown the instantaneous centre is

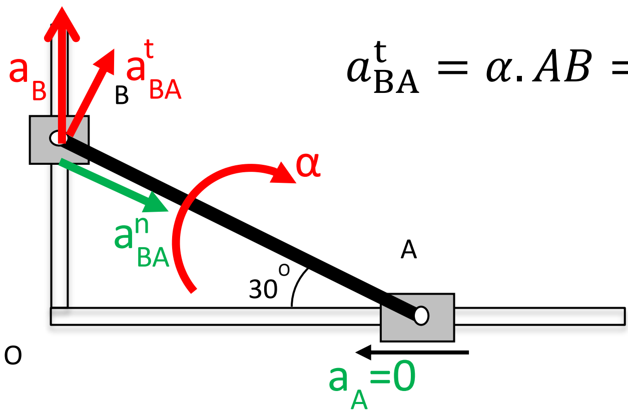



Velocity components shown

Since $V_A = r_v\omega$ then $\omega = 5/(0.5sin30)$=20 rad/s

Since $V_B = r_h\omega$ then $V_B = (0.5cos30)20$=0.866 m/s

Acceleration Analysis




$$a^{\mathrm{t}}_{\mathrm{BA}} = \alpha.AB = 0.5\,\alpha$$

$$a^{\mathrm{n}}_{\mathrm{BA}} = \omega^2.AB = 200\ \mathrm{m/s^2}$$

Acceleration components shown

The acceleration of B is calculated using vector equation

$$\underline{\boldsymbol{a}}_{\mathrm{B}} = \underline{\boldsymbol{a}}_{\mathrm{A}} + \underline{\boldsymbol{a}}^{\mathrm{n}}_{\mathrm{BA}} + \underline{\boldsymbol{a}}^{\mathrm{t}}_{\mathrm{BA}} \qquad (2)$$

Resolving Eq.(2) in horizontal direction gives

$$\overset{+}{\rightarrow}\ \Sigma X:\ \ 0 = 0 + a^{\mathrm{n}}_{\mathrm{BA}}\cos 30^{\mathrm{o}} + a^{\mathrm{t}}_{\mathrm{BA}}\sin 30^{\mathrm{o}}$$

i.e. $$0 = 0 + 200\cos 30^{o} + 0.5\alpha \sin 30^{o}$$

Hence $\alpha = -692.8\mathrm{rad/s^2}$. i.e angular acceleration is positive in the opposite direction to that shown in the figure.

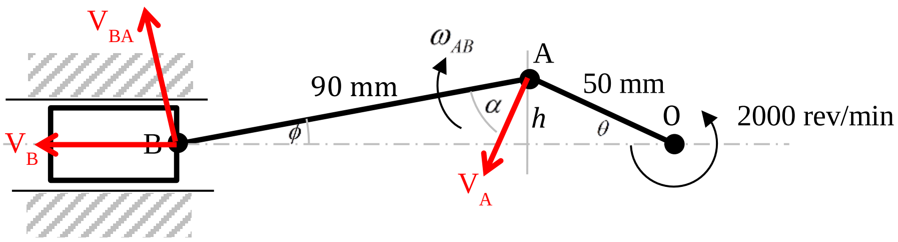
Resolving Eq.(2) in vertical direction gives

$$\uparrow^{+}\ \Sigma Y: \quad a_{\mathrm{B}} = 0 - a_{\mathrm{BA}}^{\mathrm{n}} \sin 30^{o} + a_{\mathrm{BA}}^{\mathrm{t}} \cos 30^{o}$$

i.e. $$a_{\mathrm{B}} = 0 - 200\sin 30^{o} + 0.5(-692.8)\cos 30^{o}$$

Hence $a_{\mathrm{B}} = -400\mathrm{m/s^2}$.

3)




Velocity components shown

OA=0.05 m

AB=0.09 m

$h = AB\sin\phi = OA\sin 30$ so $\phi = 16.128^\circ$ [or use sine rule]

The angular velocity of the crank is $\omega = 2000(2\pi/60) = 209.4\text{rad/s}$

The tangential velocity of A is $V_A = OA\omega = 0.05\text{x}209.4 = 10.47\text{m/s}$

The velocity of B can be calculated using vector equation:

$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

The velocity of B can be calculated easily by considering the velocity components of this equation parallel to (i.e. along) AB, i.e.

$$\swarrow \quad \underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

i.e. $$V_B \cos 16.128 = V_A \cos(\alpha) + 0$$

Noting that it can be shown that $\alpha = 43.872^\circ$, then

$$V_B = 10.47 \frac{\cos 43.87}{\cos 16.13} = 7.86\text{m/s}$$

The angular velocity of AB can be calculated by considering the velocity components of the velocity vector equation perpendicular to AB, i.e.

$$\nwarrow \quad \underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

i.e. $$V_B \sin 16.128 = -V_A \sin(\alpha) + V_{BA}$$

Re-arranging and substituting known values gives:

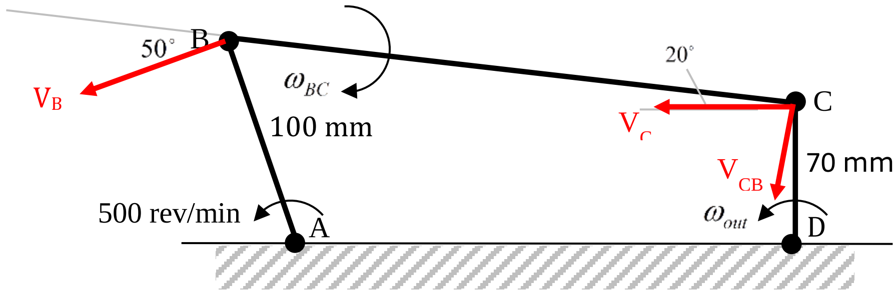
$$V_{BA} = 7.86\sin 16.13 + 10.47\sin(43.87) = 9.44\text{ m/s}$$

Noting that $V_{BA} = 0.09\omega_{AB}$, then

$$\omega_{AB} = \frac{9.44}{0.09} = 104.9\ \text{rad/s}$$

4)

Velocity Analysis




Velocity Components shown

The angular velocity of the crank is $\omega = 500(2\pi/60) = 52.4\text{rad/s}$.

The velocity of B is $V_B = AB\omega = 0.1\text{x}52.4 = 5.24\text{m/s}$

The angular velocity of the output crank can be obtained by noticing that the output crank is in pure rotation about D. This means that the velocity of C relative to D ($V_C$) is tangential to the crank, i.e: $V_C = CD\omega_{out}$, where $\omega_{out}$ is the angular velocity of the output crank.

The velocity of C (relative to fixed point A) can be calculated using the velocity vector equation:

$$\underline{v}_C = \underline{v}_B + \underline{v}_{CB}$$

Noting that $\omega_{BC}$ produces tangential velocity $V_{CB}$ and is not known, it is necessary to resolve the above equation parallel to BC to get an equation in terms of $V_C$ – this eliminates the need to consider $v_{CB}$. This gives:

$$\underline{v}_C = \underline{v}_A + \underline{v}_{CB}$$

$$V_C \cos 20 = V_B \cos 50 + 0$$

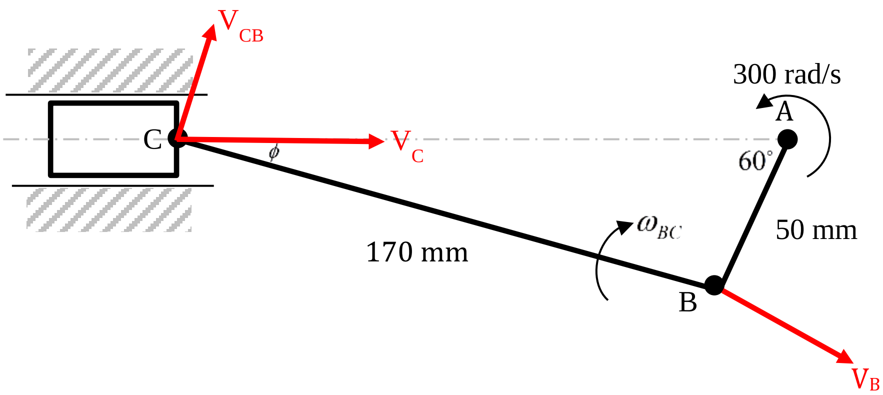
Substituting values gives:

$$V_C = \frac{5.24 \cos 50}{\cos 20} = 3.58\text{ m/s}$$

Noting from earlier that $V_C = CD\omega_{out}$, then:

$$\omega_{out} = \frac{3.58}{0.07} = 51.2\text{rad/s} = 489\text{rev/min}$$

5)




Velocity components shown

AB=0.05 m

BC=0.17 m

Using the sine rule $\dfrac{\sin\phi}{AB} = \dfrac{\sin 60}{BC}$ so $\phi = 14.757^\circ$

The angular velocity of the crank is $\omega = 300\text{rad/s}$

The tangential velocity of B is $V_B = AB\omega = 0.05\text{x}300 = 15\text{m/s}$

The angular velocity of C is needed for acceleration calculations and can be calculated using vector equation:

$$\underline{v}_C = \underline{v}_B + \underline{v}_{CB}$$

The angular velocity of BC can be calculated easily by considering the velocity components vertically, i.e.

$$\uparrow \quad \underline{v}_C = \underline{v}_B + \underline{v}_{CB}$$

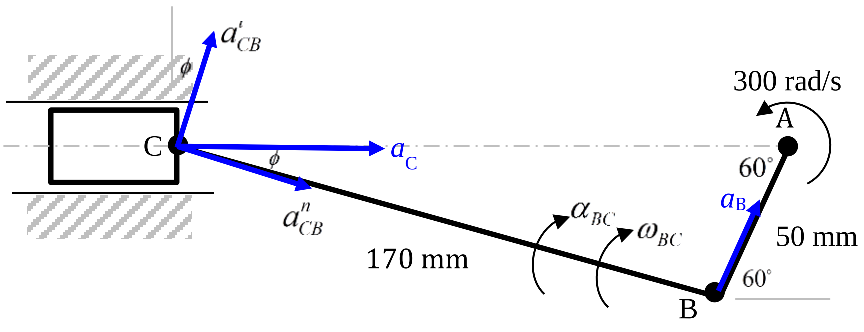
i.e. $0 = -V_B \cos 60 + V_{CB} \cos\phi$

Re-arranging and substituting gives:

$$V_{CB} = 15\frac{\cos 60}{\cos 14.757} = 7.756\text{ m/s}$$

Noting that $V_{CB} = 0.17\omega_{BC}$, then

$$\omega_{BC} = \frac{7.756}{0.17} = 45.62 \text{ rad/s}$$




Acceleration components shown

where $a_B = AB\omega^2 = 0.05\text{x}300^2 = 4500\text{m/s}^2$, $a^n_{CB} = CB\omega^2_{BC} = 0.17\text{x}45.62^2 = 353.8\text{m/s}^2$, $a^t_{CB} = CB\alpha_{BC} = 0.17\alpha_{BC}$.

The acceleration of C is calculating using acceleration vector equation

$$\underline{\boldsymbol{a}}_C = \underline{\boldsymbol{a}}_B + \underline{\boldsymbol{a}}^n_{CB} + \underline{\boldsymbol{a}}^t_{CB} \quad (2)$$

Resolving Eq.(2) in the vertical direction gives:

$$\uparrow \quad 0 = a_B \sin 60 - CB\omega^2_{BC} \sin\phi + CB\alpha_{BC} \cos\phi$$

Substituting numerical values gives:

$$0 = 4500 \sin 60 - 353.8 \sin 14.757 + \alpha_{BC} 0.17 \cos 14.757$$

It can be shown that $\alpha_{BC} = -23158\text{rad/s}^2$

Resolving Eq.(2) parallel (i.e. along) link BC (C to B +ve) gives:

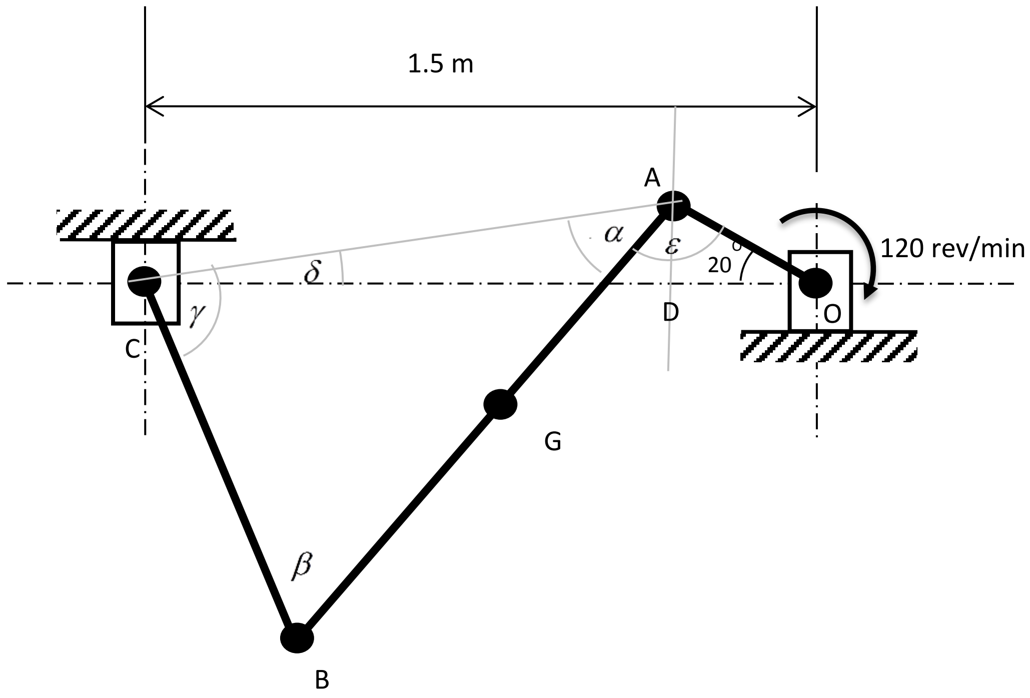
$$\searrow \quad a_C \cos\phi = a_B \cos(60+\phi) + CB\omega^2_{BC}$$

Substituting numerical values gives:

$$a_C \cos 14.757 = 4500 \cos(74.757) + 353.8$$

It can be shown that $a_C = 1589\text{m/s}^2$.

6)




OA=0.5m

AB=1.5m

BC=1.0m

CO=1.5m

AD=OAsin20=0.1710m

OD=OAcos20=0.4698m

CD=CO−OD = 1.030m

AC= $\sqrt{CD^2 + AD^2}$ =1.044 m

Using the cosine rule

$$\cos\beta = \frac{1.5^2 + 1^2 - 1.044^2}{2(1.5)(1)} = 0.7200 \text{ so } \beta = 43.94^\circ$$

$$\cos\gamma = \frac{1.044^2 + 1^2 - 1.5^2}{2(1.044)(1)} = -0.07666 \text{ so } \gamma = 94.40^\circ$$

Using the sine rule

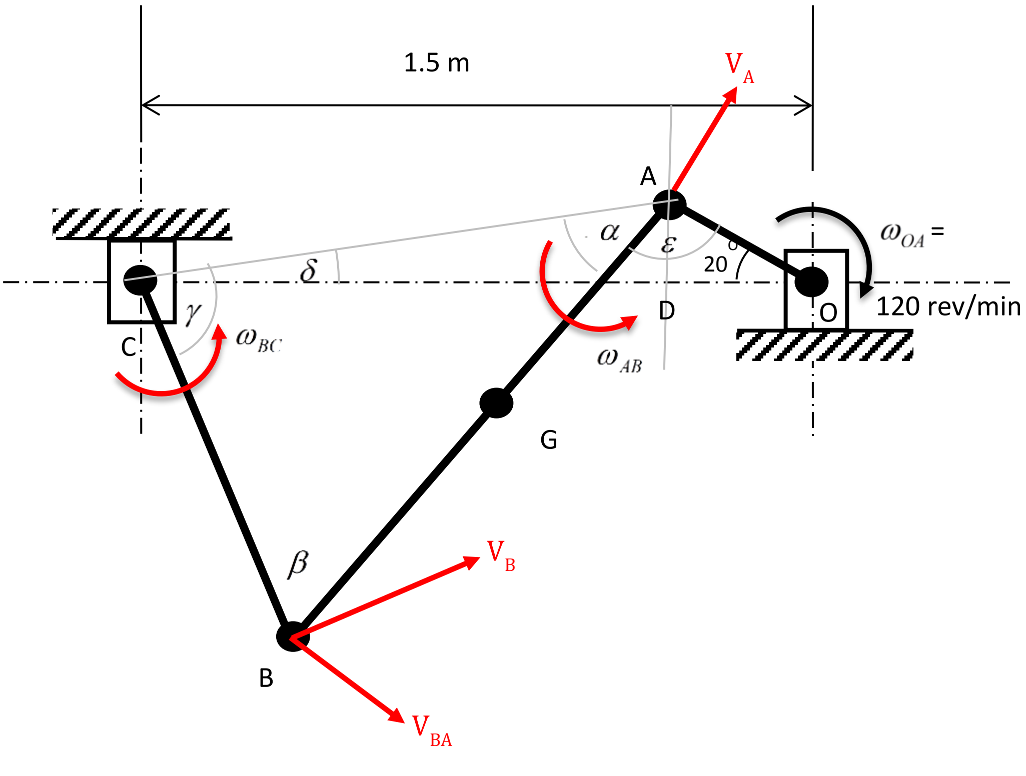
$\sin\alpha = 1.0\dfrac{\sin 43.94}{1.044}$ so $\alpha = 41.66^\circ$

Check that $\alpha + \beta + \gamma = 180$

$\tan\delta = \dfrac{AD}{CD} = \dfrac{0.1710}{1.030}$ so $\delta = 9.43^\circ$

$\varepsilon = (180 - 20 - 90) + (180 - 90 - \delta - \alpha) = 108.91^\circ$

To calculate the acceleration we need the acceleration of A and B. The acceleration of A can be calculated because OA rotates at constant angular speed, but to calculate the acceleration of B we need to calculate the angular velocity of link BA – it is also helpful to calculate the angular velocity of link BC. We start by considering velocity of B to calculate the angular velocity of link BA and the angular velocity of link BC.




Velocity Analysis

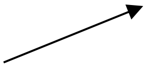
The angular velocity of the crank is $\omega_{OA} = 120(2\pi/60) = 12.57\text{rad/s}$

The tangential velocity of A is $V_A = OA\omega_{OA} = 0.5\text{x}12.57 = 6.285\text{m/s}$

The velocity of B can be determined using vector equation:

$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

Resolving this equation along AB gives:



$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

i.e. $V_A \cos(\varepsilon - 90) = V_B \cos(90 - \beta)$

Substituting values gives:

$$6.285 \cos(108.91 - 90) = V_B \cos(90 - 43.94)$$

so

$$V_B = 8.569 \text{m/s}$$

Noting that CB has fixed length and rotates about fixed point C, tangential velocity $V_B = \omega_{BC} BC$, so

$$\omega_{BC} = \frac{V_B}{BC} = \frac{8.569}{1.0} = 8.569 \text{ rad/s}$$

To calculate angular velocity $\omega_{AB}$, resolve the velocity vector equation perpendicular to AB, i.e.:

$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

i.e. $V_B \sin(90 - \beta) = -V_A \sin(\varepsilon - 90) + AB\omega_{AB}$

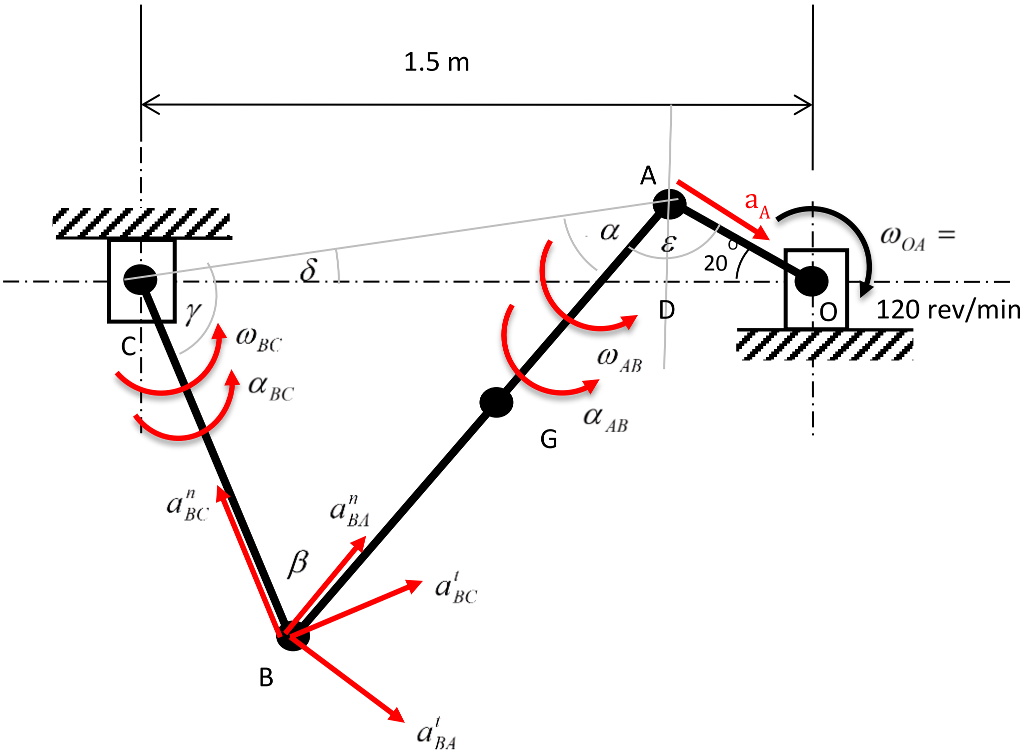
Substituting values gives:

$$8.569 \sin(90 - 43.94) = -6.285 \sin(108.91 - 90) + 1.5\omega_{AB}$$

So

$$\omega_{AB} = 5.471 \text{ rad/s}$$

Acceleration Analysis




The acceleration of B is calculating using vector equation

$$\underline{\boldsymbol{a}}_{\mathrm{B}} = \underline{\boldsymbol{a}}_{\mathrm{A}} + \underline{\boldsymbol{a}}_{\mathrm{BA}}^{\mathrm{n}} + \underline{\boldsymbol{a}}_{\mathrm{BA}}^{\mathrm{t}}$$

Noting that link BC is in pure rotation then:

$$\underline{\boldsymbol{a}}_{\mathrm{B}} = \underline{\boldsymbol{a}}_{\mathrm{BC}}^{\mathrm{n}} + \underline{\boldsymbol{a}}_{\mathrm{Bc}}^{\mathrm{t}}$$

Combining these equations gives:

$$\underline{\boldsymbol{a}}_{\mathrm{BC}}^{\mathrm{n}} + \underline{\boldsymbol{a}}_{\mathrm{Bc}}^{\mathrm{t}} = \underline{\boldsymbol{a}}_{\mathrm{A}} + \underline{\boldsymbol{a}}_{\mathrm{BA}}^{\mathrm{n}} + \underline{\boldsymbol{a}}_{\mathrm{BA}}^{\mathrm{t}}$$

where the magnitudes of each terms are

$a_{BC}^{n} = BC\omega_{BC}^{2}$ =73.43m/s$^2$

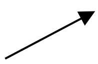
$a_{BC}^{t} = BC\alpha_{BC} = 1*\alpha_{BC} = \alpha_{BC}$

$a_{A} = OA\omega_{OA}^{2}$ =79.00 m/s$^2$

$a_{BA}^{n} = AB\omega_{BA}^{2}$ =44.80 m/s$^2$

$a_{BA}^{t} = AB\alpha_{BA} = 1.5\alpha_{BA}$

Resolving the vector equation parallel to AB gives:



$$\boldsymbol{a}_{\mathrm{BC}}^{\mathrm{n}} + \boldsymbol{a}_{\mathrm{Bc}}^{\mathrm{t}} = \boldsymbol{a}_{\mathrm{A}} + \boldsymbol{a}_{\mathrm{BA}}^{\mathrm{n}} + \boldsymbol{a}_{\mathrm{BA}}^{\mathrm{t}}$$

i.e. $$a_{BC}^{n} \cos\beta + a_{BC}^{t} \sin\beta = a_{OA} \cos(180-\varepsilon) + a_{BA}^{n}$$

Substituting values gives:

$$73.43 \cos 43.94 + \alpha_{BC} \sin 43.94 = 79.00 \cos 71.1 + 44.90$$

so

$$\alpha_{BC} = -25.39 \mathrm{rad/s}^{2}$$

(Note: direction opposite to that shown in earlier figure)

Resolving the vector equation parallel to BC gives:

$$\boldsymbol{a}_{\mathrm{BC}}^{\mathrm{n}} + \boldsymbol{a}_{\mathrm{Bc}}^{\mathrm{t}} = \boldsymbol{a}_{\mathrm{A}} + \boldsymbol{a}_{\mathrm{BA}}^{\mathrm{n}} + \boldsymbol{a}_{\mathrm{BA}}^{\mathrm{t}}$$

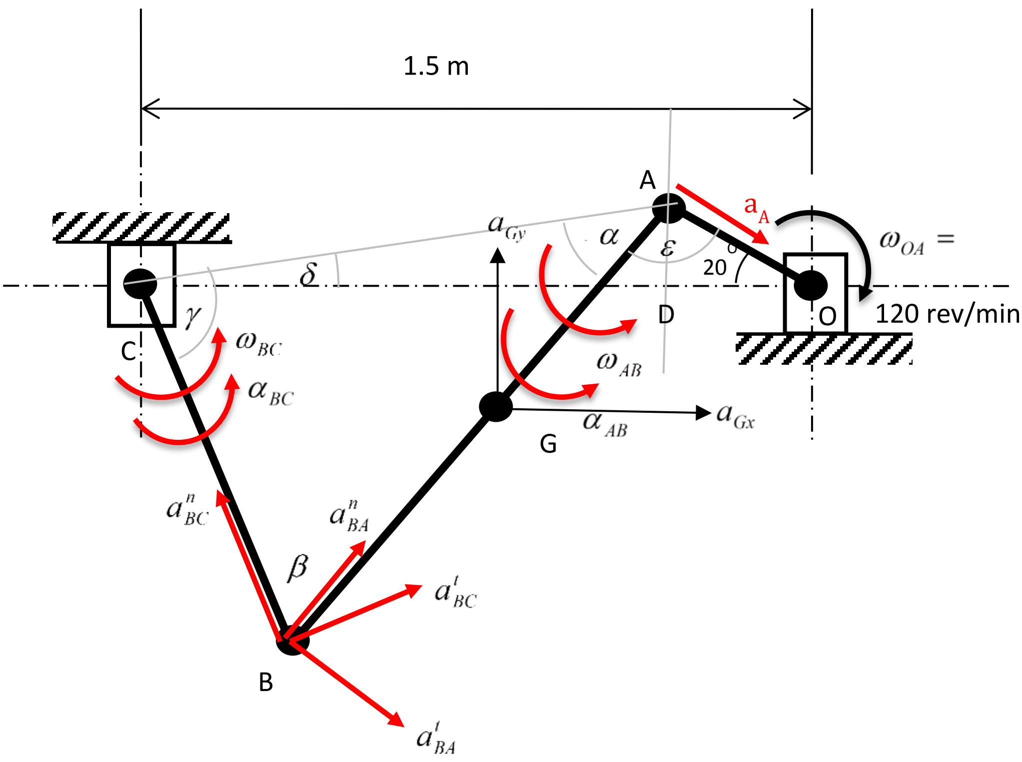
i.e. $$a_{BC}^{n} + 0 = -a_{A} \cos(\varepsilon - \beta) + a_{BA}^{n} \cos\beta - a_{BA}^{t} \sin\beta$$

Substituting values gives:

$$73.43 = -79.00 \cos 64.96 + 44.90 \cos 43.94 - 1.5\alpha_{AB} \sin 43.94$$

So

$$\alpha_{AB} = -71.61 \mathrm{rad/s}^{2}$$



The acceleration of G is calculated using vector equation:

$$\underline{a}_G = \underline{a}_A + \underline{a}_{GA}^n + \underline{a}_{GA}^t$$

where the magnitudes of each terms are:

$a_{GA}^n = 0.5a_{BA}^n$ =22.45m/s$^2$

$a_{GA}^t = 0.5a_{BA}^t$ =53.71m/s$^2$

Resolving the vector equation in the horizontal direction gives:

$$\longrightarrow \quad \underline{a}_G = \underline{a}_A + \underline{a}_{GA}^n + \underline{a}_{GA}^t$$

$$a_{Gx} = 79.00\cos 20 + 22.45\cos(180 - \varepsilon - 20) - 53.71\sin(180 - \varepsilon - 20) = 46.54\text{m/s}^2$$

Resolving the vector equation in the vertical direction gives:

$$\uparrow \qquad \underline{a}_G = \underline{a}_A + \underline{a}_{GA}^n + \underline{a}_{GA}^t$$

$$a_{Gy} = -79.00\sin 20 + 22.45\sin(180 - \varepsilon - 20) - 53.71(180 - \varepsilon - 20) = 24.17\text{m/s}^2$$

The magnitude of the acceleration of G is:

$$a_G = \sqrt{46.54^2 + 24.17^2} = 52.44\text{m/s}^2$$