Department of Mechanical, Materials and Manufacturing Engineering

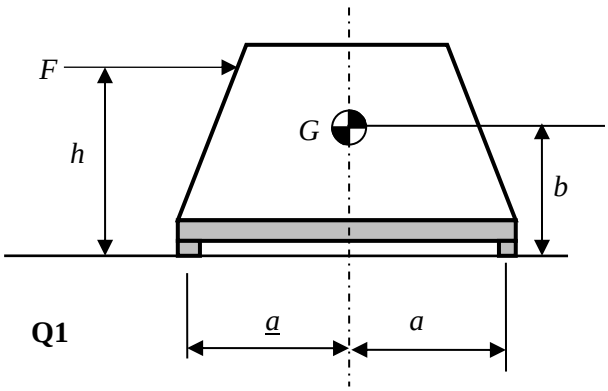
University of Nottingham

MACHINE DYNAMICS

SHEET 4: PLANAR DYNAMICS OF RIGID BODIES

1. A large object on a pallet, Figure Q1, is being slid along a floor by the application of the horizontal force *F*. The combined mass of the object and pallet is *M* and the centre of mass is at G. The pallet has one foot at the front end and one foot at the rear end, both feet extend across the full width of the pallet (into the paper). The coefficient of friction between the floor and the feet is μ. Derive expressions for the normal reactions between each foot and the floor, and the horizontal force and normal reaction at the instant when the pallet begins to tip.

$[\frac{Mg(a-\mu b)-F(h-b)}{2a}\ ;\ \frac{Mg(a+\mu b)+F(h-b)}{2a}\ ;\frac{Mg(a-\mu b)}{h-b}]$

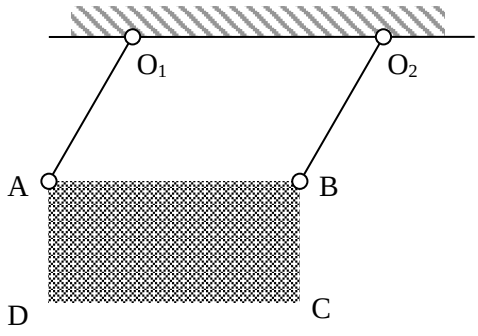


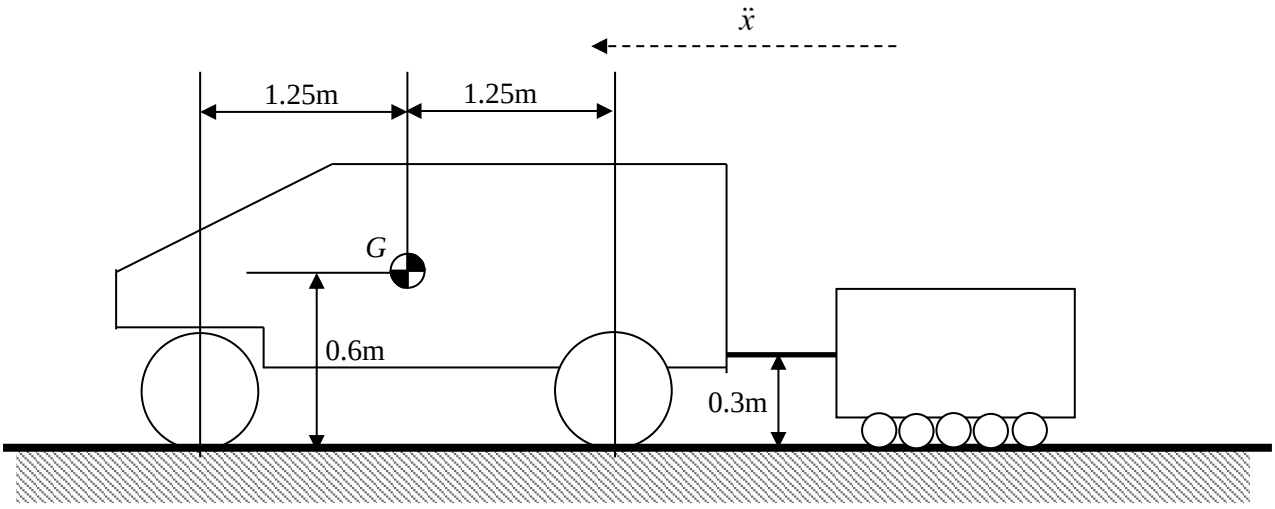
Q1

2. An object, in the form of a heavy uniform rectangular plate ABCD is used for impact testing of vehicles. It is hinged at its upper corners to two parallel rods (O_1A, O_2B) each 2 m long. These are hinged to a rigid support at their upper ends and have negligible mass. The plate can swing in the vertical plane as a bifilar pendulum such that edge *AB* remains horizontal. Find the forces in the supporting rods immediately after the plate is released from rest with the rods at 45° to the vertical. The mass of the plate is 160 kg and it has dimensions *AB*=2 m and *BC*=1 m.

[277.3 N ; 831.7 N]

Q2





Q3

3. Figure Q3 shows a *rear wheel drive* vehicle of mass 1500 kg with centre of mass at G. The vehicle is connected by a taut inextensible cable to a load of mass 500 kg that runs on rollers with negligible mass and friction. All the relevant dimensions are indicated in Figure Q3. If the coefficient of friction between the rear wheels and the ground is 0.5, determine the maximum acceleration that the vehicle and load can achieve and the corresponding normal reaction between each of the tyres and the ground. Neglect the mass of the wheels and assume that the front wheels are free to roll. Assume that changes in vehicle geometry due to suspension displacements are negligible.

[2.053 m/s^2, 4105 N at each rear tyre, 3245 N at each front tyre]

4. In a test to find the moment of inertia of the armature and shaft of a small electric motor, a mass of 2.5 kg attached to a cord wound round the 100 mm diameter shaft was found to be just sufficient to overcome the friction of the bearings. An additional mass of 3 kg was attached to the cord and allowed to fall freely from rest (with the cord initially just taut). At the end of 10.2 s the attached overall mass had fallen a distance of 2 m. If the bearing friction is assumed to remain constant, find the moment of inertia of the armature and shaft.

[1.898 kg.m^2]

5. A homogeneous rectangular plate measuring 3m x 2m with a mass of 100 kg can rotate in a vertical plane about a horizontal, frictionless hinge O attached to the midpoint of the shorter side. If it is released from rest with OG horizontal (G is the centre of mass of the plate) find the angular acceleration of the plate and the reaction force at the hinge immediately after release.

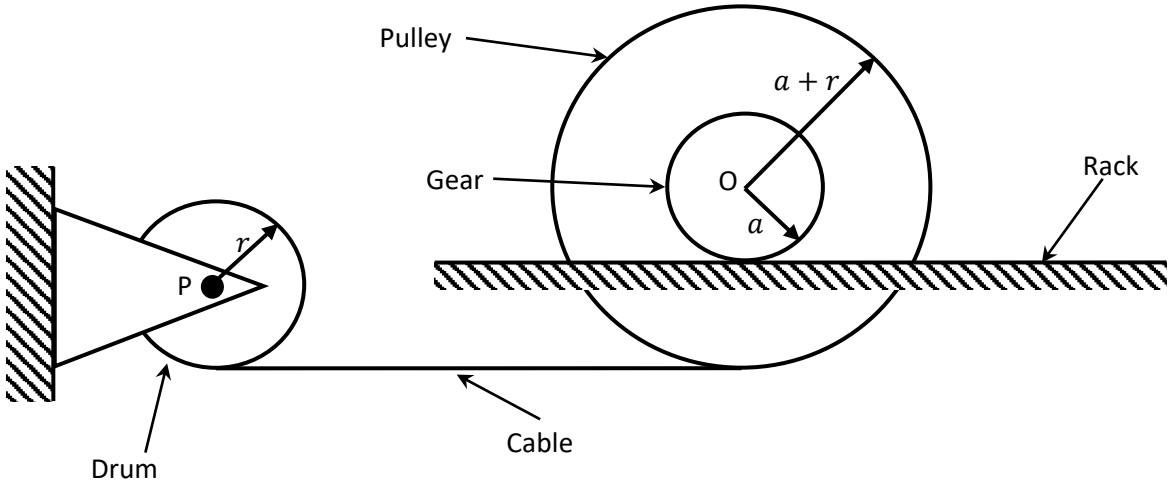
[4.410 rad/s^2; 318.5 N (vertically upwards)]

6. A uniform cylindrical tube of mass *M* starts to roll without sliding from rest down a slope inclined at an angle α. The inner and outer radii of the tube are a and b. Find the time *T* taken for the tube to roll a distance *L* down the plane. (The moment of inertia for objects can be found in the MM1DMS notes).

$[T = \sqrt{\frac{L(a^2+3b^2)}{gb^2 \sin\alpha}}]$

7. A drum of radius r rotating about fixed point P is used to wind up the cable that is wrapped around the pulley of radius $a + r$, centred at O. A gear of radius a, also centred at O, is integrally attached to the pulley and rolls on a fixed toothed rack as shown. The drum has moment of inertia J_P about its axis. The pulley and gear have mass *M* and moment of inertia J_O about the axis through O. If a torque *L* is applied to the drum, find the horizontal acceleration of the gear centre O.

$[\frac{aL}{J_O+J_P+Ma^2}]$

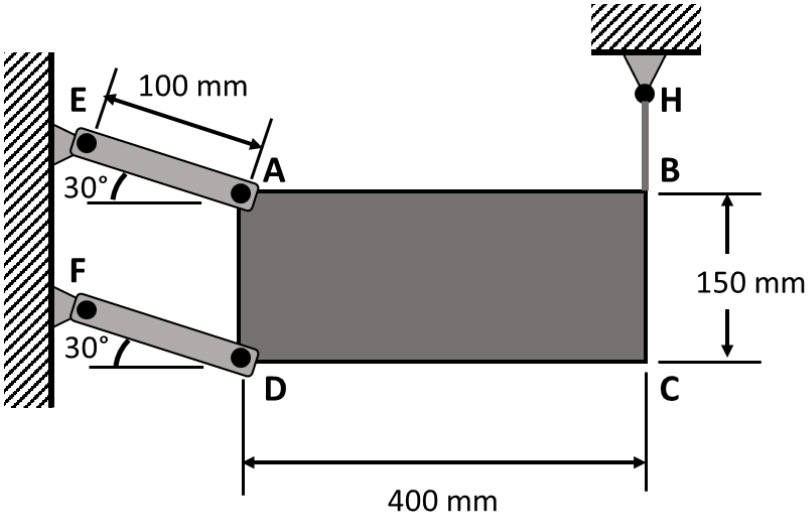


8. Revisit the bar linkage mechanism shown in Question 3 of Exercise Sheet 3. If the mass of link AO is 5 kg, the mass of link AB is 20 kg, and the mass of link BC is 10kg, find the reaction forces at O and A. Treat each link as a uniform bar with centre of mass at their midpoint.

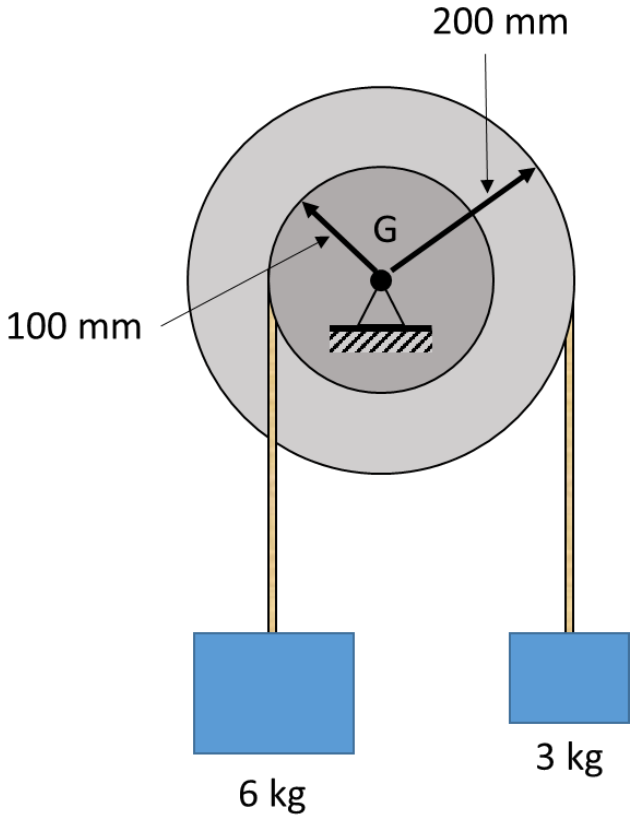
9. The thin homogenous plate ABCD of mass m=5 kg is held in place as shown below by the wire HB. Neglecting the mass of the links EA and FD, determine immediately after the wire has been cut:

(a) The acceleration of the centre of mass of the plate. [8.496 m/s2]

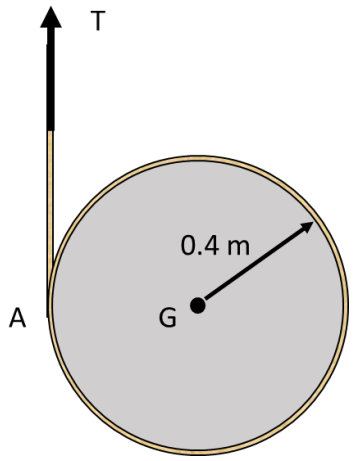
(b) The force carried by each one of the links EA and FD. [16.52 N, -6.62 N]



10. The pulley below is connected to two blocks as shown. The total moment of inertia of the system around G is equal to I_G=0.3 kgm^2. Assuming a frictionless system, determine the angular acceleration of the pulley and the acceleration of each block. [0 m/s2, 0 m/s2].



11. A cord is wrapped around a homogeneous disk of mass 10 kg. The cord is pulled upwards with a force T =200 N as shown below.



Determine: (a) The acceleration of the center of the disk, [10.19 m/s^2]

(b) The angular acceleration of the disk, [-40 rad/s^2]

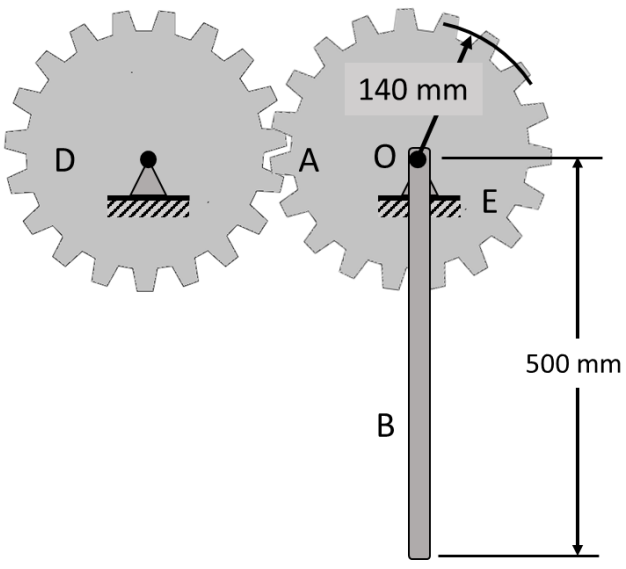
(c) The acceleration of the cord. [26.19 m/s^2]

12. In the system shown below the rod OB is rigidly connected to gear E at point O and revolves along with it. The entire portion EOB of the mechanism is actuated by gear D and at the instant shown has a clockwise angular velocity of 8 rad/s and a counterclockwise angular acceleration of 30 rad/s^2. It is given that the mass of the rod AB is m_{OB}=4 kg, the mass of the gear E is m_E=6 kg and the moment of inertia of gear E around O is 0.04 kgm^2.

Determine:

a) The tangential force exerted by gear D. Consider no normal force component at the contact point of the two gears.

b) The components of the reaction at shaft O.

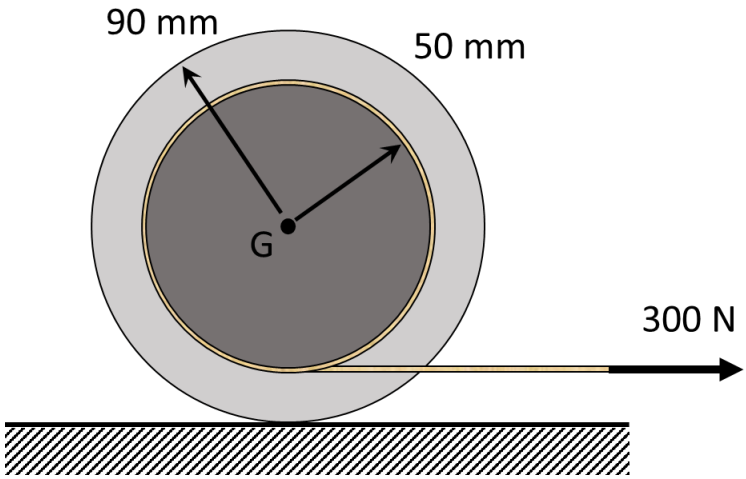


13. A cord is wrapped around the inner hub of a wheel and pulled horizontally with a force of 300 N as shown below. The wheel has a mass of 30 kg and a moment of

inertia around G equal to $I_G=0.3\ kgm^2$. Knowing that the static friction coefficient between the wheel and the road surface is $\mu_s=0.3$ and the dynamic friction coefficient is $\mu_k=0.2$, determine:

a) The acceleration of the centre of mass G and

b) The angular acceleration of the wheel.


90 mm
50 mm
G
300 N