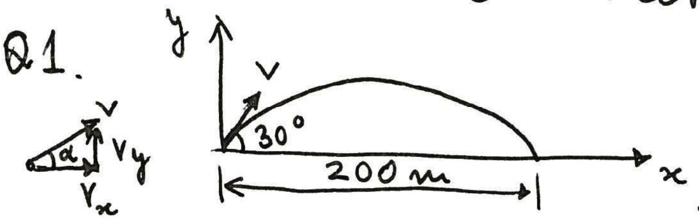
Exercise Sheet 1

Q1.



$m\ddot{x} = 0 \rightarrow \ddot{x} = 0$

$m\ddot{y} = -g \rightarrow \ddot{y} = -g$

$\dot{x} = V_x \qquad \dot{y} = -gt + V_y$

$x = V_x t \qquad y = -g\frac{t^2}{2} + V_y t$

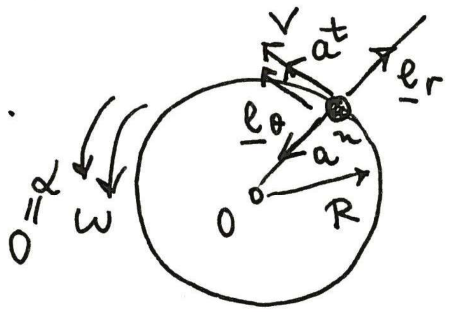
$t = \frac{x}{V_x} \qquad V_y / V_x = \tan\alpha$

$$y = -\frac{g}{2}\frac{x^2}{V_x^2} + \frac{V_y}{V_x}x = -\frac{gx^2}{2(V\cos\alpha)^2} + x.\tan\alpha$$

$$0 = -\frac{g\,x}{2V^2\cos^2\alpha} + \frac{\sin\alpha}{\cos\alpha} : \quad V^2 = \frac{gx}{2\sin\alpha\cos\alpha} = \frac{gx}{\sin 2\alpha}$$

$$V = \sqrt{\frac{200 \times 9.80}{\sin 60°}} = 47.57 \text{ m/s} \qquad t = \frac{200}{V\cos 30°} = 4.855 \text{ s}$$

Q2.



$\underline{V} = R\omega\,\underline{e}_\theta$

$\underline{a} = \underline{a}^n + \underline{a}^t = -\omega^2 R\,\underline{e}_r + \overset{0}{\alpha} R\,\underline{e}_\theta$

$\underline{a} = -\omega^2 R\,\underline{e}_r$ for $\omega = \text{const.}$

$V = R\omega = 10 \times 2 = 20 \text{ m/s}$

$a = \omega^2 R = 2^2 \times 10 = 40 \text{ m/s}^2$

Q3) Initial position is when *t=0*

$$r = i$$

Remembering that $\ddot{r} = acceleration$

$$\dot{r} = 3i + 4tj$$

$$\ddot{r} = 4j$$

Since the acceleration vector does not depend on *t* it is said to be constant.

Q4) The position vector is found by integrating the velocity vector.

$$r = i\int 4tdt + j\int 5t^2dt + C_1i + C_2j$$

$$r = 2t^2i + \frac{5}{3}t^3j + C_1i + C_2j$$

At time t = 0, r = 5*i* – 6*j*

$$C_1 = 5, C_2 = -6$$

Therefore:

$$r = (2t^2 + 5)i + \left(\frac{5}{3}t^3 - 6\right)j$$

Q5) a)

$$r = \frac{9}{2}t^2i + \frac{8}{5}t^{\frac{5}{2}}j$$

$$v = 9ti + 4t^{\frac{3}{2}}j$$

When t = 4

$$Velocity = (36i + 32j)ms^{-1}$$

b)

$$\frac{dv}{dt} = 9i + 6t^{\frac{1}{2}}j$$

When t = 4

$$a = 9i + 12j$$

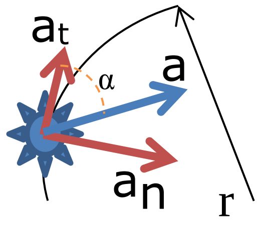
Therefore:

$$|a| = \sqrt{(9^2 + 12^2)}$$

$$|a| = 15ms^{-2}$$

Q6) In order to work with SI units:

90 km/h=90000/3600=25 m/s
72 km/h=72000/3600=20 m/s



The average tangential acceleration of the automobile will be equal to:

$$a_t = \frac{\Delta v}{\Delta t} = \frac{20 - 25}{8} = -0.625\ m/s^2$$

Immediately after the brakes have been applied the velocity of the automobile has not changed therefore

$$a_n = \frac{v^2}{r} = \frac{25^2}{750} = 0.833\ m/s^2$$

The direction of the total acceleration can be calculated as

$$\tan(\alpha) = \frac{a_n}{a_t} = -1.3328$$

$$\alpha = 53.1^o$$

While the magnitude of the acceleration will be equal to

$$a = ({a_n}^2 + {a_t}^2)^{1/2} = 1.041\ \text{m/s}^2$$