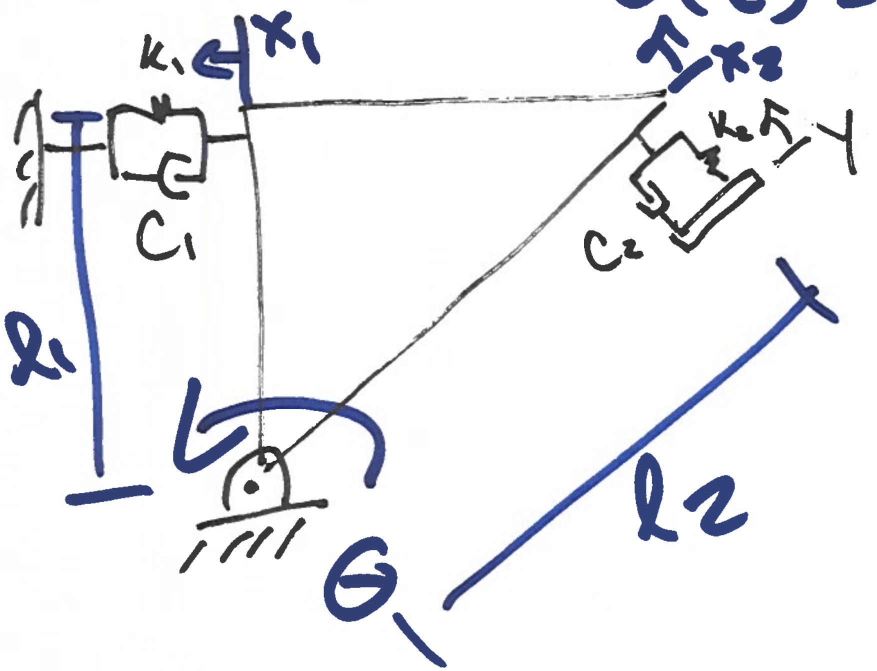
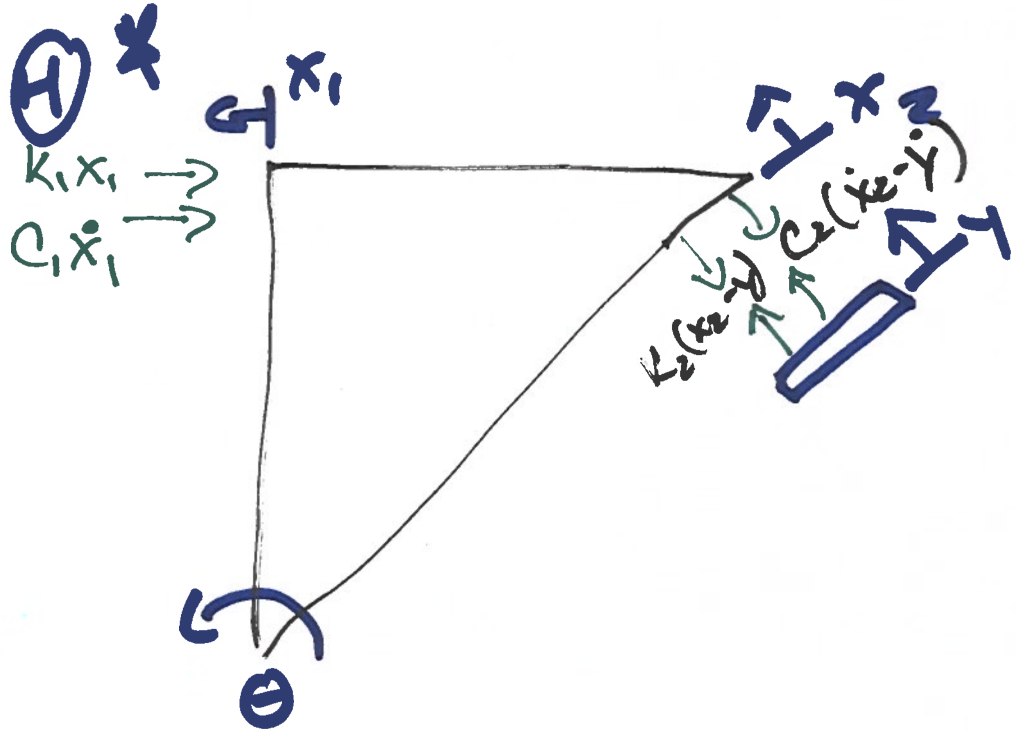
ES 5 Q3

$$\theta(t) = |\Theta^*| \sin(\omega t + \alpha)$$

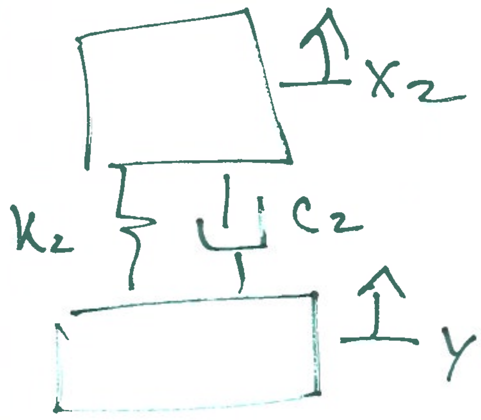




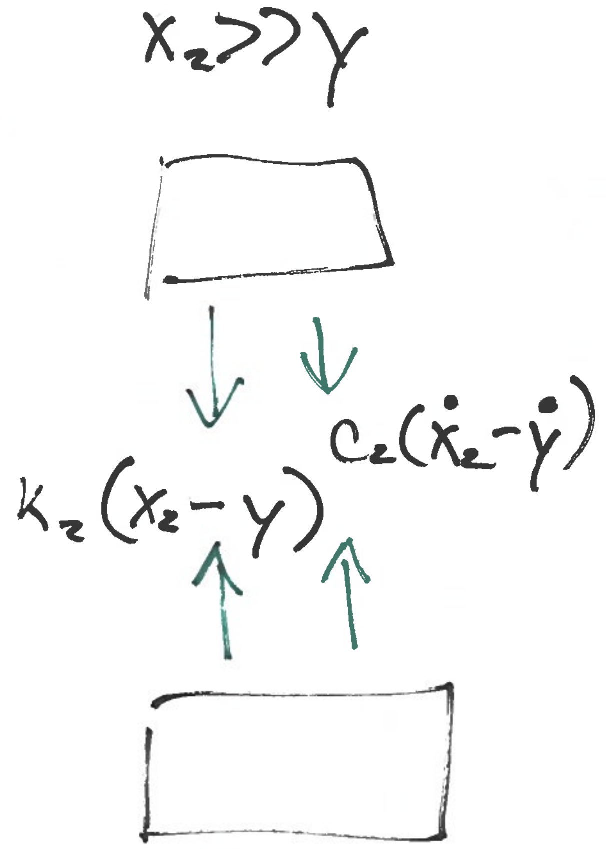
$x_1 = l_1 \theta$ $\qquad$ $x_2 = l_2 \theta$

$\dot{x}_1 = l_1 \dot{\theta}$ $\qquad$ $\dot{x}_2 = l_2 \dot{\theta}$

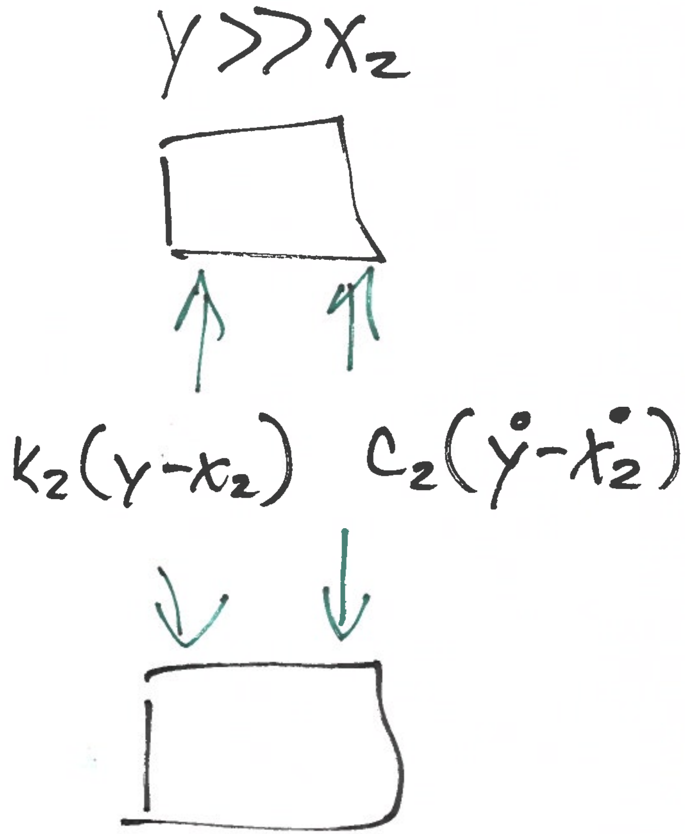
$\ddot{x}_1 = l_1 \ddot{\theta}$ $\qquad$ $\ddot{x}_2 = l_2 \ddot{\theta}$



x_2
k_2
c_2
y



$x_2 >> y$
$c_2(\dot{x}_2 - \dot{y})$
$k_2(x_2 - y)$



$y >> x_2$
$k_2(y - x_2)$
$c_2(\dot{y} - \dot{x}_2)$

$$\Sigma T_0 = I_0 \ddot{\theta}$$

$$-l_1 k_1 x_1 - l_1 c_1 \dot{x}_1 - l_2 k_2 (x_2 - y) - l_2 c_2 (\dot{x}_2 - \dot{y}) = I_0 \ddot{\theta}$$

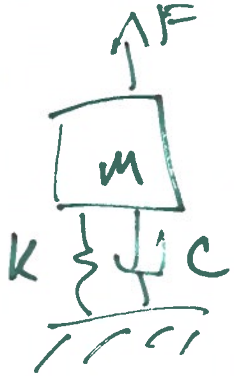
$$-l_1 k_1 l_1 \theta - l_1 c_1 l_1 \dot{\theta}_1 - l_2 k_2 (l_2 \theta - y) - l_2 c_2 (l_2 \dot{\theta} - \dot{y}) = I_0 \ddot{\theta}$$

$$\underbrace{\boxed{I_0}}_{M} \ddot{\theta} + \underbrace{\boxed{(c_1 l_1^2 + c_2 l_2^2)}}_{C} \dot{\theta} + \underbrace{\boxed{(k_1 l_1^2 + k_2 l_2^2)}}_{K} \theta = \underbrace{\boxed{k_2 l_2}}_{K_r} y + \underbrace{\boxed{c_2 l_2}}_{C_R} \dot{y}$$

H^*

$$H(\omega) = \frac{Z^*}{F} = \frac{H^*}{T}$$

$$= \frac{Z^*}{R} = \frac{H^*}{R}$$

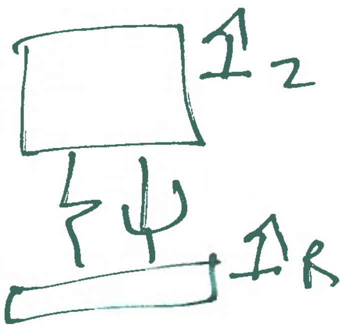


$$M\ddot{z} + C\dot{z} + kz = 0$$

$$M\ddot{z} + C\dot{z} + kz = f(t) = F \sin \omega t$$

$$\cos$$

$$e^{\omega t}$$



$$M\ddot{z} + C\dot{z} + kz = k_r\, r(t) + C_r\, \dot{r}(t)$$

$$H(\omega) = \frac{H^*}{\cancel{A}} = \frac{k_r + C_r \omega i}{(k - M\omega^2) + C\omega i} = \frac{k_2 l_2 + C_2 l_2 \omega i}{\left((k_1 l_1^2 + k_2 l_2^2) - I_0 \omega^2\right) + i(C_1 l_1^2 + C_2 l_2^2)}$$

$$I_0 \ddot{\Theta} + (C_1 l_1^2 + C_2 l_2^2)\dot{\Theta} + (k_1 l_1^2 + k_2 l_2^2)\Theta = k_2 l_2 y + C_2 l_2 \dot{y}$$

$$\Theta^*$$

$$y = \bar{Y} e^{i\omega t}$$

$$\dot{y} = \bar{Y} i\omega e^{i\omega t}$$

$$\Theta = \Theta^* e^{i\omega t}$$

$$\dot{\Theta} = \Theta^* i\omega e^{i\omega t}$$

$$\ddot{\Theta} = -\Theta^* \omega^2 e^{i\omega t}$$

$$-I_0 \omega^2 \Theta^* \cancel{e^{i\omega t}} + (C_1 l_1^2 + C_2 l_2^2) i\omega \Theta^* \cancel{e^{i\omega t}} + (k_1 l_1^2 + k_2 l_2^2) \Theta^* \cancel{e^{i\omega t}}$$

$$= k_2 l_2 \bar{Y} \cancel{e^{i\omega t}} + C_2 l_2 \bar{Y} i\omega \cancel{e^{i\omega t}}$$

$$\left[I_0 \omega^2 + (C_1 l_1^2 + C_2 l_2^2) i\omega + k_1 l_1^2 + k_2 l_2^2 \right] \Theta^* = \left[k_2 l_2 + C_2 l_2 i\omega \right] \bar{Y}$$

$$H(\omega) = \frac{\Theta^*}{\text{[illegible]}} = \frac{\bar{Y}(k_2 l_2 + C_2 l_2 i\omega)}{I_0 \omega^2 + k_1 l_1^2 + k_2 l_2^2 + (C_1 l_1^2 + C_2 l_2^2) i\omega}$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{k_1 l_1^2 + k_2 l_2^2}{I_0}}$$

$$\xi = \frac{C}{2\sqrt{KM}} = \frac{C_1 l_1^2 + C_2 l_2^2}{2\sqrt{(k_1 l_1^2 + k_2 l_2^2) I_0}}$$

$$\omega_d = \Omega_n = \omega_n \sqrt{1 - \xi^2}$$

$$H(\omega) = \frac{c + di}{e + fi}$$

$$|H^*| = \sqrt{\frac{c^2 + d^2}{e^2 + f^2}}$$

Unfortunately, the complex number expressions you will derive are not generally in the form

$$Z^* = a + \mathbf{i}\, b$$

The general case for you is the form

$$Z^* = \frac{c + \mathbf{i}\, d}{e + \mathbf{i} f}$$

Finding the amplitude of this ratio of complex numbers is easy. It's just

$$Z = \left|Z^*\right| = \sqrt{\frac{c^2 + d^2}{e^2 + f^2}}$$

Finding the phase lag is more difficult, since we first need to convert the expression into the form

$$Z^* = a + \mathbf{i}\, b$$

To do this, we multiply both numerator and denominator by the complex conjugate of the denominator. That is,

$$Z^* = \frac{c + \mathbf{i}\,d}{e + \mathbf{i}\,f} \times \frac{e - \mathbf{i}\,f}{e - \mathbf{i}\,f}$$

$$Z^* = a + \mathbf{i}\,b = \left(\frac{ce + df}{e^2 + f^2}\right) + \mathbf{i}\left(\frac{de\text{-}cf}{e^2 + f^2}\right)$$

$$\boxed{a = \left(\frac{ce + df}{e^2 + f^2}\right) \qquad b = \left(\frac{de\text{-}cf}{e^2 + f^2}\right)}$$

The phase angle is then given by $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$ as before.

So for our problem...

$$Z^* = \frac{F}{\left(K - M\omega^2\right) + \mathbf{i}\omega C}$$

where

$$c = F$$
$$d = 0$$
$$e = K - M\omega^2$$
$$f = \omega C$$

$$Z = \left|Z^*\right| = \frac{c}{\sqrt{e^2 + f^2}} = \frac{F}{\sqrt{\left(K - M\omega^2\right)^2 + \omega^2 C^2}}$$

Same as before!!