The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2020-2021

DYNAMICS AND CONTROL

Time allowed THREE hours plus 30 minutes upload period

Open-book take-home examination

Answer ALL questions

You must submit a single pdf document, produced in accordance with the guidelines provided on take-home examinations, that contains all of the work that you wish to have marked for this open-book examination. Your submission file should be named in the format '[Student ID]_MMME2046.pdf'.

Write your student ID number at the top of each page of your answers.

This work must be carried out and submitted as described on the Moodle page for this module. All work must be submitted via Moodle by the submission deadline. **Work submitted after the deadline will not be accepted without a valid EC.**

No academic enquiries will be answered by staff and no amendments to papers will be issued during the examination. If you believe there is a misprint, note it in your submission but answer the question as written.
Contact your Module Teams Channel or SS-AssessEng-UPE@exmail.nottingham.ac.uk for support as indicated in your training.

Plagiarism, false authorship and collusion are serious academic offences as defined in the University's Academic Misconduct Policy and will be dealt with in accordance with the University's Academic Misconduct Procedures. The work submitted by students must be their own and you must declare that you understand the meaning of academic misconduct and have not engaged in it during the production of your work.

ADDITIONAL MATERIAL: Formula Sheet

1. Fig.Q1 shows a crankshaft-piston mechanism which is driven by an electric motor inputting a torque T at point A. At the instant shown $\gamma=15^o$ and $\delta=45^o$ and link AB is rotating clockwise at constant angular velocity ω_{AB}=33rad/s. The piston has mass 0.3kg and can be modelled as a particle. Gravitational acceleration is g=9.81m/sec^2. Link AB has mass m_{AB}=0.1kg while link BC has negligible mass. The friction coefficient between the piston and the chamber walls μ=0.1. Lengths AB=40mm and BC=150mm.

(a) State the number of Degrees of Freedom for the mechanism. [2]

(b) Determine the velocity of point B and indicate its direction on a diagram. [2]

(c) Draw a velocity diagram for link BC and determine the angular velocity ω_{BC} for rod BC. [5]

(d) Employing d'Alembert's principle sketch clear free-body diagrams for AB, BC and piston C, taking care to show all forces (including pin reaction forces and inertia forces). [6]

(e) Determine the acceleration of point B and indicate its direction on a diagram. [4]

(f) Draw an acceleration diagram for link BC and determine the acceleration of point C. [5]

(g) Determine the force applied by link BC to the piston at point C. [6]

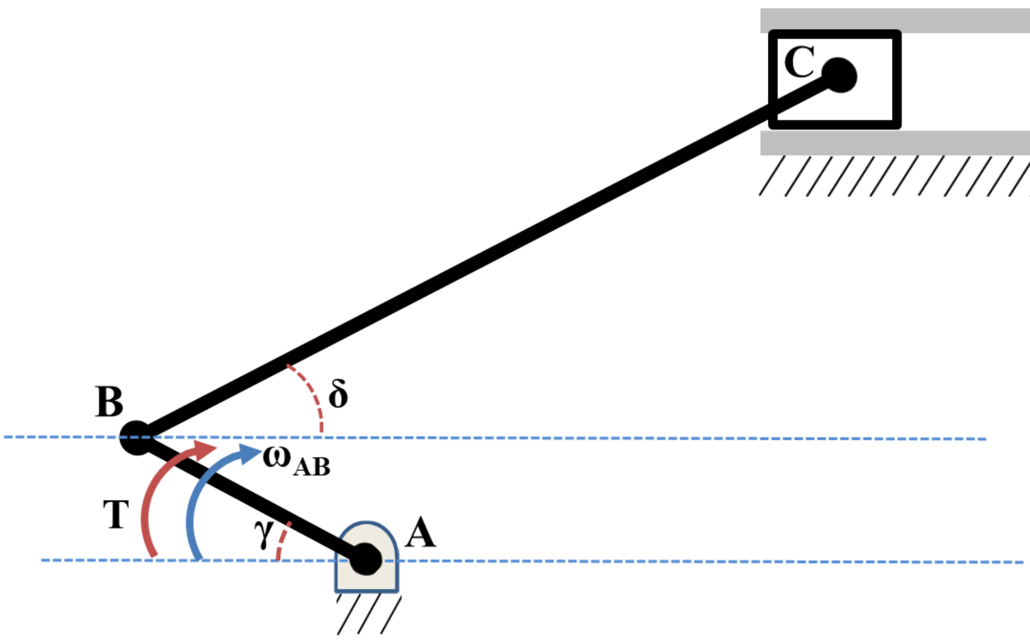

FIGURE Q1

2. Figure Q2.1 shows the plant part of a control system.

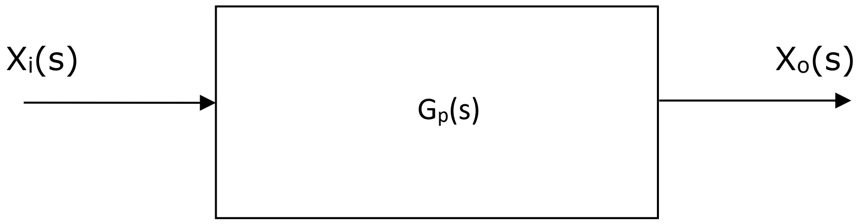

FIGURE Q2.1

If the plant is subjected to a unit step input at t=0 and $G_p(s) = \frac{1}{4s+10}$, determine:

(a) The steady state error as $t \to \infty$. [3]

(b) The time taken for the system to be at 95% of the final (steady state) level. [7]

The plant is integrated with a feedback controller as shown in Figure Q2.2.

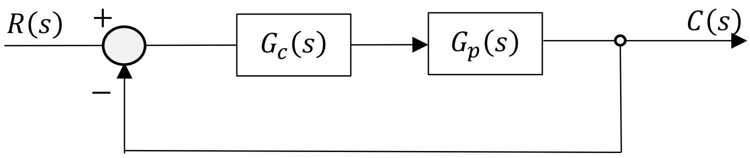

FIGURE Q2.2

(c) Determine the transfer function $\frac{C(s)}{R(s)}$ of the system in terms of $G_c(s)$ and $G_p(s)$. [4]

(d) If $G_c(s) = 3s + k$**,** calculate the value of k that reduces the steady state error to 5% of the value achieved when the system is subject to a unit step input. [4]

To improve the system response further, a PID controller is implemented for which:

$$G_c(s) = \frac{3s^2 + ks + 2}{s}$$

(e) What is the effect on the steady state error for a unit step input for k= 10? [4]

(f) What is the range of values of *k* for which the system will be stable? [4]

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(g) Figure Q2.3 shows the root locus for a 2nd order system with an overall transfer function given by:

$$G(s) = \frac{\omega^2}{s^2 + 2\gamma\omega s + \omega^2}$$

Describe the system behaviours you would expect to see in response to a step input for the roots shown at *a*, *b*, *c* and *d* in the graph. [4]

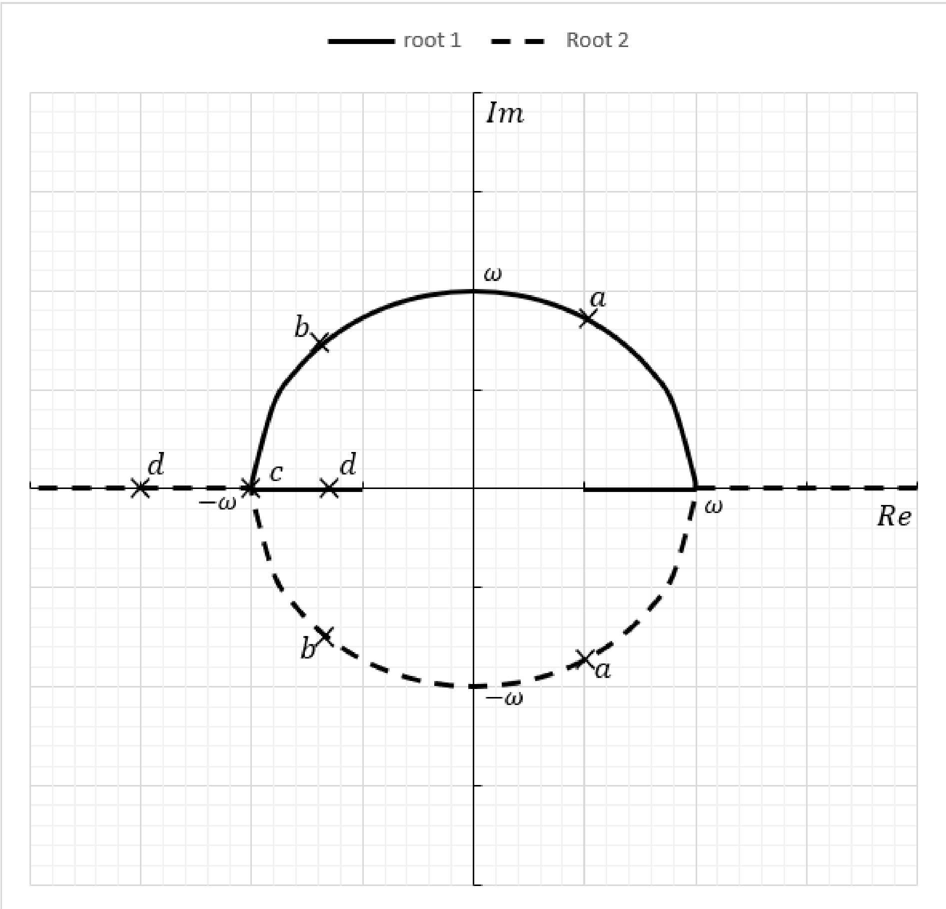

Figure Q2.3

3. Figure Q3.1 shows a rigid beam AD with moment of inertia I_o about the pivot at point B. The beam is supported at D by a vertical spring having stiffness k and at A by a damper having damping coefficient c. θ denotes the angular displacement of the beam about the horizontal equilibrium state and is assumed positive in the clockwise direction as shown. All displacements and rotations in this system are assumed to be small.

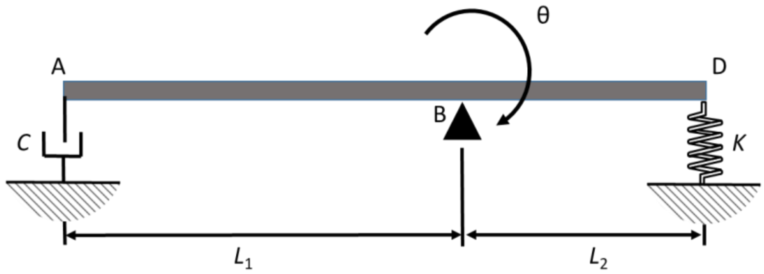

FIGURE Q3.1

I_o = 0.5 kg m^2
k = 1000 N/m
c = 10 Ns/m
L_1 = 1 m
L_2 = 0.5 m

(a) Draw the Free Body Diagram for the rigid beam. [3]

(b) Derive the equation of motion governing rotation of the beam $\theta(t)$ about pivot B. [3]

(c) Calculate the undamped natural frequency ω_n and damping ratio ζ for the system. [4]

(d) If the beam is lifted vertically up by 0.1m at point A and then released from rest determine the resulting transient angular displacement at B as a function of time, $\theta_{tr}(t)$. What frequency will this system vibrate at after being released from rest? [8]

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A harmonic moment, $M(t) = Me^{i\omega t} = 10e^{18.9it}$, is now applied at point B as shown in Figure Q3.2 and the system allowed to come to steady state motion.

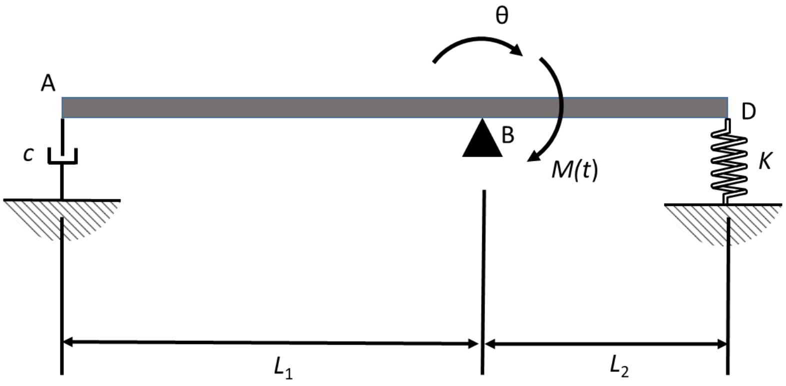

FIGURE Q3.2

(e) Derive the frequency response function $\frac{\Theta^*}{M}$ describing rotation of the beam under excitation moment $M(t) = Me^{i\omega t}$. [4]

(f) Calculate the magnitude of the steady state response, $|\theta^*|$, and its phase relative to the applied moment, α. E.g. determine $\theta_{ss}(t) = |\theta^*|cos(18.9t + \alpha)$. [4]

(g) Determine the amplitude of the force acting on the ground, $|Q^*|$, through the spring, *k*, at steady state. [8]

(h) If the system is experiencing unwanted levels of motion when operating at a given frequency, describe one way you might reduce the amplitude of motion at this frequency and explain one drawback of the method you have chosen. [6]

END