The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS & MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2021-2022

DYNAMICS AND CONTROL

Time allowed THREE HOURS

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL questions

Only a calculator from approved list B may be used in this examination.

Basic Models	***Scientific Calculators***	***Graphical Calculators***
Aurora HC133 Casio HS-5D Deli – DL1654 Sharp EL-233	Aurora AX-582 Casio FX82 family Casio FX83 family Casio FX85 family Casio FX350 family Casio FX570 family Casio FX 991 family Sharp EL-531 family Texas Instruments TI-30 family Texas BA II+ family	Casio FX9750 family Texas Instruments TI82 family

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: Formula Sheet

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

MACHINE DYNAMICS

(5 points Q1 & 25 points Q2; 30 POINTS TOTAL)

1. FIGURE Q1 shows a rigid bar AB, of length L, that slides down an incline. At the instant shown end A is sliding along a horizontal plane and end B is sliding along an inclined plane ($\beta = 60°$).

(a) How many degrees of freedom does the system have at the instant shown? [1]

(b) Find the angle α at the moment when ends A and B have equal speed. [4]

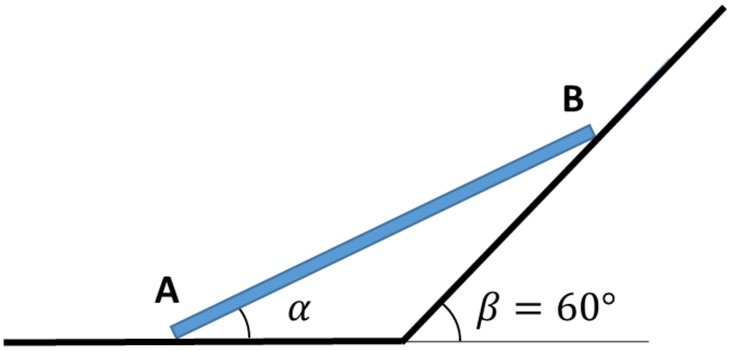

FIGURE Q1

2. FIGURE Q2 shows a four-link mechanism (a crane mechanism) with a load $F=10^4$ N applied to point C in the vertical direction. Points O_1 and O_2 are fixed. The lengths of the links are: O_1A = 15 m, AB = 5m, O_2B=13.61 m. Link O_1A rotates with constant angular velocity ω_1=2 rad/s. Link ABC has mass concentrated along edge AB. The centre of mass of link ABC is located at point G and AG=2.5 m. The mass of link ABC is 1000 kg, and its moment of inertia is 5000 kg.m^2.

(a) Find the velocity of point C at the instant shown. [5]

(b) Find the acceleration of point G at the instant shown. [5]

(c) Perform a dynamic force analysis on the mechanism at the instant shown by using D'Alembert's Principle:

i) Draw the free body diagram of the mechanism; [5]

ii) Find the torque applied to the link O_1A required to drive the mechanism. Ignore gravitational forces in all links and assume that links O_1A and O_2B are massless; [5]

iii) Find the reaction forces at points O_1 and O_2. [5]

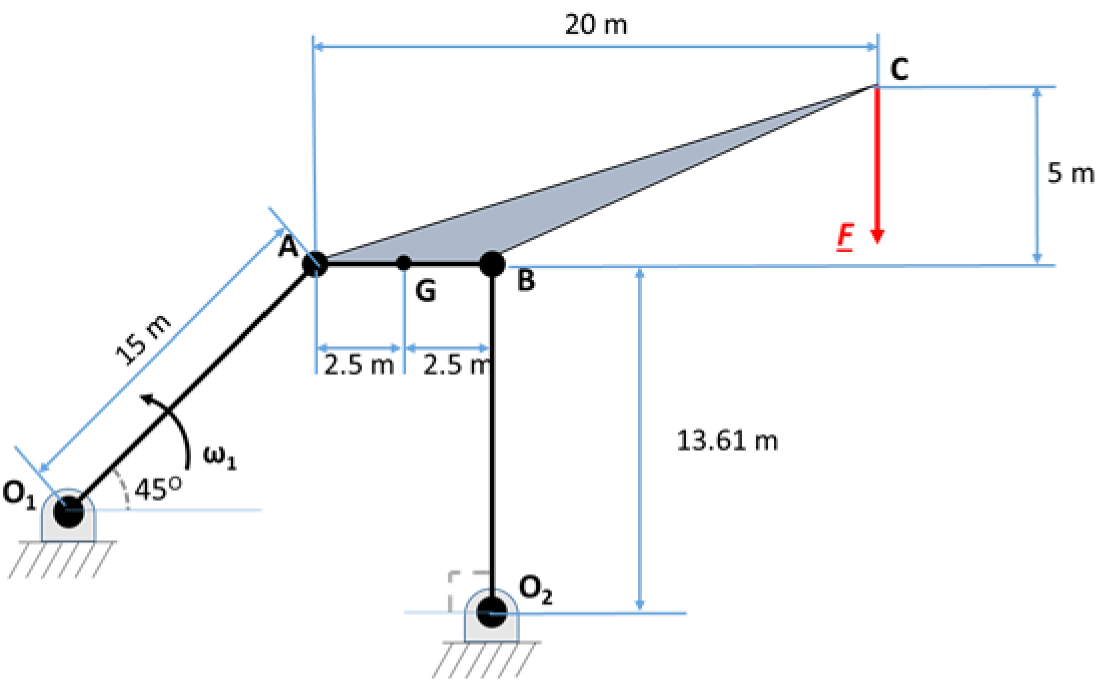

FIGURE Q2

CONTROLS

(30 POINTS)

3. FIGURE Q3 shows a system consisting of a controller with transfer function $G_c(s)$, plant with transfer function $G_c(s)$, and feedback loop with a constant proportional gain k.

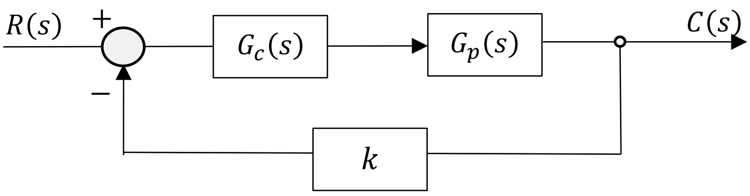

FIGURE Q3

For the system shown in FIGURE Q3 determine:

(a) The open loop transfer function $G(s) = {C(s)}/{R(s)}$ [1]

(b) The closed loop transfer function [4]

The system in FIGURE Q3 is then subjected to a unit step input $R(s) = \frac{1}{s}$ at t=0.

For:

$$G_p(s) = \frac{1}{(3s^2 + 10s + 8)}$$

$$G_c(s) = 3s + 4$$

$$k = 1$$

Determine:

(c) The steady state error as $t \to \infty$. [5]

(d) The value of $C(t)$ at t=0.2s. [5]

To eliminate steady state error the controller, $G_c(s)$, is replaced with a controller using the following transfer function:

$$G_c(s) = \frac{s^2 + s + n}{s}$$

(e) What is the effect on steady state error for a unit step input if $n = 1$, while the plant transfer function and k remain unchanged? [5]

(f) Derive the closed loop transfer function for this system, and determine the Routh Array associated with this system. At what values of n will the system be stable? [10]

VIBRATIONS

(17 points Q4 & 23 points Q5; 40 POINTS TOTAL)

4. A coupling is made between two rotational shafts that can be considered equivalent to the model shown in FIGURE Q4. Assume the shafts have mass moments of inertia I_1 and I_2 as shown, and have equal radii, *r*. The stiffness and damping elements in the couplings are equivalent to *k* and *c* respectively with:

$$r = 0.1\text{m}$$
$$k = 100\ \text{N/m}$$
$$c = 1\ \text{Ns/m}$$
$$I_1 = 10\ \text{kg.m}^2$$
$$I_2 = 20\ \text{kg.m}^2$$

(a) Draw a fully annotated Free Body Diagram for each of the rotational elements in terms of their rotational displacements, θ_1 and θ_2, for the positive directions of motion shown in Figure Q4. [6]

(b) Derive the equations of motion for each of the rotational elements. [4]

(c) Determine the generalized matrix, $[Z]\{\Theta\} = \{0\}$ for this system. [4]

(d) Calculate the undamped natural frequencies associated with this system. [3]

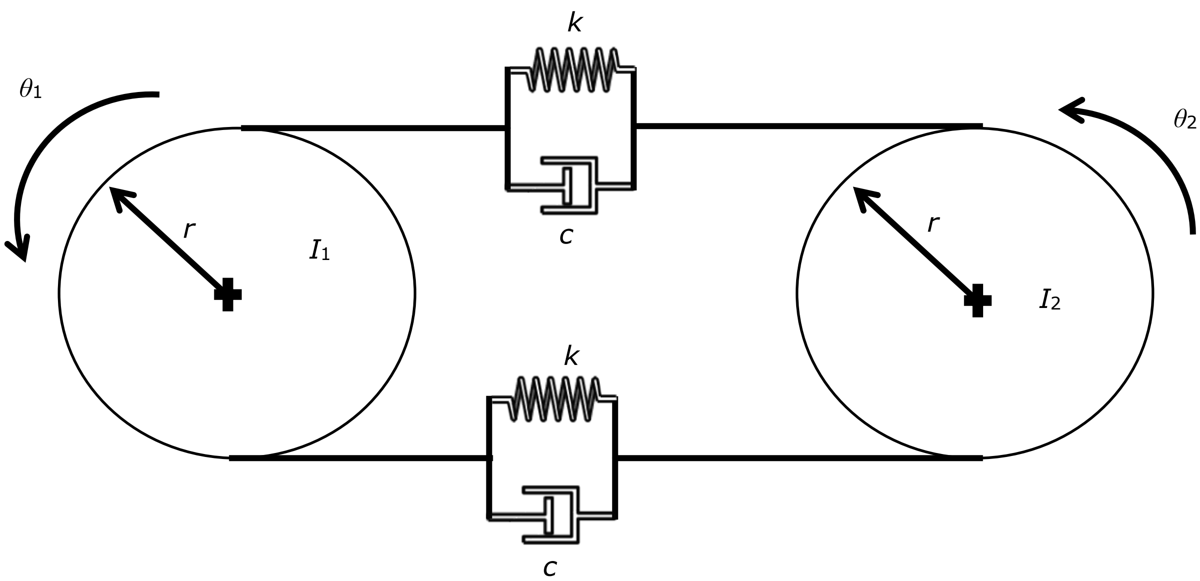

Figure Q4

5. FIGURE Q5 shows a Single Degree of Freedom dynamic mass-spring-damper system acting through pivot, P, with damping and spring elements as shown. Data values are:

$$I_p = 10 \text{ kg m}^2$$
$$K_t = 100{,}000 \text{ N/m}$$
$$C_t = 50 \text{ Ns/m}$$
$$K_s = 20{,}000 \text{ N/m}$$
$$C_s = 1{,}000 \text{ Ns/m}$$
$$L_1 = 0.7 \text{ m}$$
$$L_2 = 0.8 \text{ m}$$

(a) Draw a fully annotated Free Body Diagram for the system. [3]

(b) Derive the Equation of Motion for the system in terms of rotational displacement, θ. [3]

(c) Determine the undamped natural frequency. [2]

(d) Determine the damping ratio. [2]

(e) If the system is allowed to vibrate freely, what frequency will this occur at? [2]

(f) The beam is given a push at time, t=0 s, resulting in an initial clockwise angular velocity of $\dot{\theta}_i = 6$ rad/s, from its initial rest position, $\theta_i = 0$ rad. Predict the maximum angular displacement of the axle in the subsequent free vibration. You may find the exact solution, or an approximation. If you use an approximation state any assumptions used. [11]

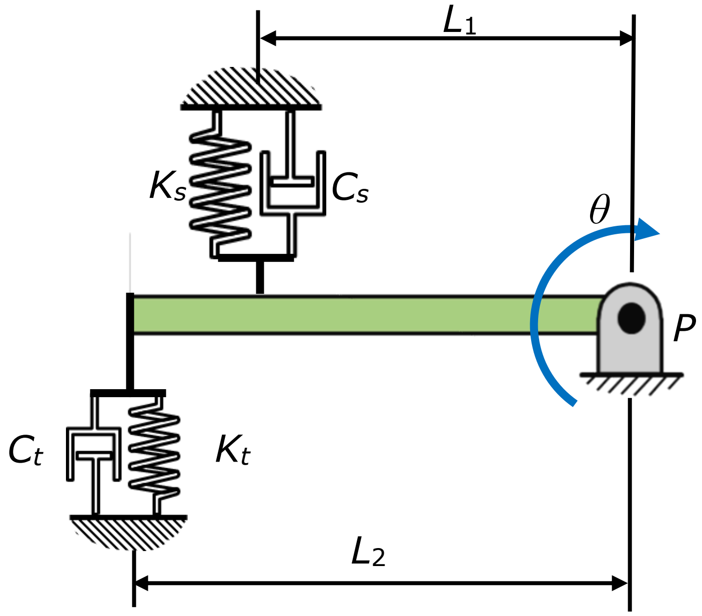

Figure Q5

END