Basic thermodynamic relations

$$n = \frac{m}{\tilde{m}}$$

$$pV = n\tilde{R}T$$

$$\tilde{R} = 8314 \text{ J/kmol}$$

$$R = \frac{\tilde{R}}{\tilde{m}}$$

$$pv = RT$$

$$pV = mRT$$

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

$$H_2 - H_1 = mc_p(T_2 - T_1)$$

$$H = U + pV$$

$$u_2 - u_1 = c_v(T_2 - T_1)$$

$$h_2 - h_1 = c_p(T_2 - T_1)$$

$$h = u + pv$$

for a cyclic process:

$$W_{net} + Q_{net} = 0$$

Closed: $W + Q = U_2 - U_1$
SFEE: $\dot{Q} + \dot{W} = \dot{m}(h_2 - h_1) + \dot{m}\left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right) + \dot{m}g(z_2 - z_1)$

$$q + w = (h_2 - h_1) + \left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right) + g(z_2 - z_1)$$

$$s_2 - s_1 = \int_1^2 \left[\frac{dQ}{T}\right]_{reversible}$$

$$\Delta S_{total} = \Delta S_{system} + \Delta S_{surrounding}$$

$$\eta = \frac{net\ work}{heat\ supplied} = \left|\frac{W}{Q_{hot}}\right|$$

$$\eta_{carnot} = 1 - \frac{Q_{cold}}{Q_{hot}}$$

$$\eta_{carnot} = 1 - \frac{T_{cold}}{T_{hot}}$$

$$\oint \frac{dQ}{T} = 0 \text{ reversible}$$

$$\oint \frac{dQ}{T} < 0 \text{ irreversible}$$

Relations for perfect gases

$$pV = mRT$$

$$c_p - c_v = R$$

$$\frac{c_p}{c_v} = \gamma$$

$$s_2 - s_1 = c_v ln\frac{T_2}{T_1} + Rln\frac{v_2}{v_1}$$

$$s_2 - s_1 = c_p ln\frac{T_2}{T_1} - Rln\frac{p_2}{p_1}$$

$$p_1v_1 = p_2v_2 = \text{const}$$

$$p_1{v_1}^{\gamma} = p_2{v_2}^{\gamma} = \text{const}$$

$$\frac{p_2}{p_1} = \left[\frac{V_1}{V_2}\right]^{\gamma}$$

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_1}{T_2} = \left[\frac{V_1}{V_2}\right]^{\gamma-1}$$

Formulae for work

Closed processes

$$w = -\int_1^2 pdv \text{ general}$$

$$w = -p[v_2 - v_1] \text{ isobaric}$$

$$w = 0 \quad \text{isochoric}$$

$$W = -mRTln\left[\frac{v_2}{v_1}\right] \text{ isothermal}$$

$$w = \frac{-(p_2v_2 - p_1v_1)}{1-n} \text{ polytropic}$$

Open processes

$$\dot{W} = \dot{m}\int_1^2 vdp \text{ general}$$

$$\dot{W} = \dot{m}v[p_2 - p_1] \text{ isochoric}$$

$$\dot{W} = \dot{m}RTln\left[\frac{p_2}{p_1}\right] \text{ isothermal}$$

$$\dot{W} = \dot{m}\frac{n}{n-1}[p_2v_2 - p_1v_1]$$

or

$$\dot{W} = \dot{m}R\frac{n}{n-1}[T_2 - T_1]$$

polytropic

Compressors

$$\eta_{isothermal} = \frac{W_{isothermal}}{W_{polytropic}}$$

$$\eta_{volumetric} = \frac{V_{induced}}{V_{swept}}$$

$$\eta_{vol} = 1 - \frac{V_C}{V_S}\left\{\left[\frac{p_2}{p_1}\right]^{\frac{1}{n}} - 1\right\}$$

$$\dot{Q}_{jacket} = \dot{m}\frac{\gamma - n}{1 - n}c_v(T_2 - T_1)$$

$$p_i = \sqrt{p_1p_2}$$

$$\dot{m} = N\rho\eta_{vol}V_{swept}$$

Gas mixtures and air condition

$$p = \sum p_i$$

$$U = \sum U_i$$

$$c_v = \sum \frac{m_i}{m}c_{v,i}$$

$$c_p = \sum \frac{m_i}{m}c_{p,i}$$

$$R = \sum \frac{m_i}{m}R_i$$

$$\frac{1}{\tilde{m}} = \sum \frac{m_i}{m}\frac{1}{\tilde{m}_i}$$

$$\frac{V_i}{V} = \frac{m_i}{m}\frac{\tilde{m}}{\tilde{m}_i}$$

Thermodynamics formulae

$$\frac{1}{R} = \sum \frac{n_i}{n} \frac{1}{R_i}$$

$$\tilde{m} = \sum \frac{n_i}{n} \tilde{m}_i$$

$$\frac{V_i}{V} = \frac{p_i}{p} = \frac{n_i}{n}$$

$$\omega = \frac{mass\ of\ water\ vapour, m_s}{mass\ of\ air, m_a}$$

$$\omega = 0.622 \frac{p_s}{(p - p_s)}$$

$$\phi = \frac{p_s}{p_g}$$

$$T_{dew\ point} = T_{sat\ at\ P_g}$$

$$CoP = \frac{useful\ heat\ transfer}{compressor\ work}$$

Combustion gases

Molar masses of common combustion species:

gas	Molar mass, kg/kmol
O_2	32
N_2	28
CO_2	44
CO	28
H_2	2
H_2O	18
C	12

Vapour power cycles

$$\eta = \frac{net\ work\ out}{external\ heat\ in}$$

$$SSC = \frac{3600}{net\ specific\ power\ out}$$

$$r_w = \frac{net\ work\ out}{gross\ work\ out}$$

$$\eta_{isen.turbine} = \frac{actual\ work\ out}{isentropic\ work\ out}$$

$$\eta_{isen.compress} = \frac{isentropic\ work\ in}{actual\ work\ in}$$

Heat transfer

$$\dot{Q} = kA\frac{\Delta T}{\Delta x}$$

$$\dot{Q} = \frac{\Delta T}{R_{th}}$$

$R_{th} = \frac{\Delta x}{kA}$ (conduction)

$R_{th} = \frac{1}{hA}$ (convection)

$R_{th} = \frac{ln\frac{r_o}{r_i}}{2\pi kL}$ (axisymmetric)

$$UA = \frac{1}{R_{th}}$$

$$\dot{Q} = hA(T_f - T_w)$$

$$Nu_d = \frac{hd}{k}$$

$$Pr = \frac{c_p\mu}{k}$$

$$Gr = \frac{g\beta L^3\rho^2\Delta T}{\mu^2}$$

Heat exchangers

$$\dot{Q} = UAf\Delta T_{mean}$$

f is correction factor

U is overall heat transfer coeff't

$$\Delta T_{mean} = \frac{\Delta T_a - \Delta T_b}{ln\,{}^{\Delta T_a}/_{\Delta T_b}}$$

$$\dot{Q} = \dot{m}_h c_{p.h}(T_{hi} - T_{hout})$$

$$= \dot{m}_c c_{p.c}(T_{ci} - T_{cout})$$

$$NTU = \frac{UA}{(\dot{m}c_p)_{min}}$$

$$\varepsilon = \frac{actual\ heat\ transfer\ rate}{maximum\ heat\ transfer\ rate}$$

$$= \frac{\dot{q}}{\dot{q}_{max}}$$

$$\dot{q}_{max} = C_{min}\Delta T_{max}$$

in the case $\dot{m}_h c_{p,h} < \dot{m}_c c_{p,c}$ then:

$$\varepsilon = \frac{(T_{hot-in} - T_{hot-out})}{(T_{hot-in} - T_{cold-in})}$$

in the case $\dot{m}_h c_{p,h} > \dot{m}_c c_{p,c}$ then:

$$\varepsilon = \frac{(T_{cold-out} - T_{cold-in})}{(T_{hot-in} - T_{cold-in})}$$

NAVIER-STOKES EQUATIONS

2D continuity equation, incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2D momentum equation, incompressible flow:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Shear stress:

$$\tau = \mu\frac{du}{dy}$$

BOUNDARY LAYERS

Displacement thickness: $\boldsymbol{\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy}$

Displacement thickness: $\boldsymbol{\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy}$

Momentum thickness: $\boldsymbol{\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$

Surface shear:

$$\boldsymbol{\tau_0 = \rho\, U^2 \frac{d\theta}{dx}}$$

or equivalent $\boldsymbol{\tau_0 = \frac{1}{2}\rho\, U^2 C_f}$

Drag force (Single flat plate)

$$\boldsymbol{D = \rho\, b\, U^2\, \theta}$$

or equivalent $\boldsymbol{D = \frac{1}{2}\rho\, U^2 \mathrm{bL}\, C_D}$

Laminar boundary layer velocity profile (von Karman approximation):

$$\frac{u}{U} = \frac{y}{\delta}\left(2 - \frac{y}{\delta}\right)$$

Turbulent boundary layer velocity profile (1/7th power law approximation):

$$\frac{u}{U} \approx \left(\frac{y}{\delta}\right)^{1/7}$$

Smooth flat plate boundary layers		
Quantity	Laminar flow Blasius exact solution	Turbulent flow Prandtl Approximate Solution
Boundary layer thickness, δ	$\frac{\delta}{x} \approx \frac{5}{Re_x^{0.5}}$	$\frac{\delta}{x} \approx \frac{0.16}{Re_x^{1/7}}$
Displacement thickness, δ^*	$\frac{\delta^*}{x} = \frac{1.721}{Re_x^{0.5}}$	$\frac{\delta^*}{x} \approx \frac{0.02}{Re_x^{1/7}}$
Momentum thickness, θ	$\frac{\theta}{x} = \frac{0.664}{Re_x^{1/2}}$	$\frac{\theta}{x} = \frac{0.0156}{Re_x^{1/7}}$
Shape factor, H	2.59	1.28
Skin friction coefficient, C_f	$C_f = \frac{0.664}{Re_x^{0.5}}$	$C_f \approx \frac{0.027}{Re_x^{1/7}}$
Drag coefficient, C_D	$C_D = \frac{1.328}{Re_x^{1/2}}$	$C_D = \frac{0.031}{Re_x^{1/7}}$

$$Re_x = \frac{\rho U x}{\mu}$$

Drag coefficient for fully-rough flat plate boundary layer:

$$C_D = \left(1.89 + 1.62\, log\, \frac{L}{\varepsilon}\right)^{-2.5}$$

LIFT AND DRAG

Lift coefficient $C_L = \frac{L/A_p}{\frac{1}{2}\rho V^2}$

Drag coefficient: $C_D = \frac{D/A}{\frac{1}{2}\rho U^2}$

Aspect ratio: $A_{Ratio} = \frac{b^2}{A_p}$

Increase in angle of attack: $\Delta\alpha = \frac{C_L}{\pi A_{Ratio}}$

Drag coefficient for finite aerofoil:

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi A_{Ratio}}$$

Stall speed : $U_{min} = \sqrt{\frac{2W}{\rho C_{L max} A}}$

Safe minimum landing speed (FAA): $U = 1.2 U_{min}$

Spin ratio: $Spin\ ratio = \frac{\omega R}{V} = \frac{\omega D}{2V}$

Fluids formulae

COMPRESSIBLE FLOW

Mach number: $Ma = \frac{v}{a}$

Speed of sound in a perfect gas: $a = \sqrt{\gamma RT}$

Stagnation enthalpy: $h + \frac{1}{2}v^2 = h_o$

Stagnation temperature: $T_o = T + \frac{v^2}{2C_p}$

Stagnation properties, perfect gas:

$$\frac{T_o}{T} = 1 + \frac{v^2}{a^2}\left(\frac{\gamma-1}{2}\right) = 1 + Ma^2\left(\frac{\gamma-1}{2}\right)$$

$$\frac{a_o}{a} = \left[1 + Ma^2\left(\frac{\gamma-1}{2}\right)\right]^{0.5}$$

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left\{1 + Ma^2\left(\frac{\gamma-1}{2}\right)\right\}^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}} = \left\{1 + Ma^2\left(\frac{\gamma-1}{2}\right)\right\}^{\frac{1}{\gamma-1}}$$

Isentropic perfect gas:

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

Critical properties

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{T^*}{T_o} = \frac{2}{\gamma+1}$$

$$\frac{a^*}{a_o} = \left(\frac{2}{\gamma+1}\right)^{0.5}$$

$$v^* = a^* = \sqrt{\gamma RT^*}$$

Isentropic duct flow

$$\frac{\rho^*}{\rho} = \left\{\left(\frac{2}{\gamma+1}\right)\left[1 + Ma^2\left(\frac{\gamma-1}{2}\right)\right]\right\}^{\frac{1}{\gamma-1}}$$

$$\frac{v^*}{v} = \frac{1}{Ma}\left\{\left(\frac{2}{\gamma+1}\right)\left[1 + Ma^2\left(\frac{\gamma-1}{2}\right)\right]\right\}^{0.5}$$

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\frac{1 + \frac{(\gamma-1)}{2}Ma^2}{\frac{(\gamma+1)}{2}}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$\dot{m}_{max} = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} \rho_o A^* \sqrt{RT_O}$$

Normal shock

$$\frac{\rho_2}{\rho_1} = \frac{1 + \beta\frac{p_2}{p_1}}{\beta + \frac{p_2}{p_1}} \text{ where } \beta = \frac{\gamma+1}{\gamma-1}$$

$$\frac{p_2}{p_1} = \frac{1}{\gamma+1}\left[2\gamma Ma_1^2 - (\gamma-1)\right]$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma Ma_1^2}{1 + \gamma Ma_2^2}$$

$$Ma_2^2 = \frac{(\gamma-1)Ma_1^2 + 2}{2\gamma Ma_1^2 - (\gamma-1)}$$

$$\frac{T_2}{T_1} = \left[2 + (\gamma-1)Ma_1^2\right]\frac{2\gamma Ma_1^2 - (\gamma-1)}{(\gamma+1)^2 Ma_1^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{(\gamma+1)Ma_1^2}{(\gamma-1)Ma_1^2 + 2}$$

Mach cone: $\sin\alpha = \frac{1}{Ma}$

TURBOMACHINERY

Bernoulli equation:

$$\left(\frac{1}{2g}u^2 + \frac{p}{\rho g} + z\right) = H_T$$

Power delivered to fluid: $P_w = \rho g Q H$

$$H = H_{T(outlet)} - H_{T(inlet)}$$

Power to drive pump: $P = \omega T$

Pump efficiency: $\eta = \frac{P_w}{P} = \frac{\rho g Q H}{\omega T}$

Total efficiency: $\eta = \eta_v \eta_h \eta_m$

Net positive suction head: $NPSH = \frac{p_i}{\rho g} + \frac{v_i^2}{2g} - \frac{p_v}{\rho g}$

$$C_Q = \frac{Q}{nD^3}$$

$$C_H = \frac{gH}{n^2 D^2}$$

$$C_P = \frac{P}{\rho n^3 D^5}$$

$$\eta = \frac{C_H C_Q}{C_P}$$

Pump similarity:

$$C_{Q1} = C_{Q2}; \quad \frac{Q_2}{Q_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3$$

$$C_{H1} = C_{H2}; \quad \frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{D_2}{D_1}\right)^2$$

$$C_{P1} = C_{P2}; \quad \frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{n_2}{n_1}\right)^3\left(\frac{D_2}{D_1}\right)^5$$

$$\eta_1 = \eta_2$$

Specific speed: $N_S' = \frac{(C_Q^*)^{\frac{1}{2}}}{(C_H^*)^{\frac{3}{4}}} = \frac{nQ^{*\frac{1}{2}}}{(gH^*)^{\frac{3}{4}}}$

where * denotes BEP

Parameter	Definition
Reynolds number	$Re = \frac{\rho U L}{\mu}$
Mach number	$Ma = \frac{v}{a}$
Froude number	$Fr = \frac{U^2}{gL}$
Weber number	$We = \frac{\rho U^2 L}{\gamma}$
Specific heat ratio	$\gamma = \frac{C_p}{C_v}$
Roughness number	$\frac{\epsilon}{L}$
Lift coefficient	$C_L = \frac{L/A_p}{\frac{1}{2}\rho V^2}$
Drag coefficient	$C_D = \frac{D/A}{\frac{1}{2}\rho U^2}$
Strouhal Number	$St = \frac{fD}{U}$
Grashof number	$Gr = \frac{g\beta L^3 \rho^2 \Delta T}{\mu^2}$

DIMENSIONAL ANALYSIS

Quantity	Dimensions
Length	L
Area	L^2
Volume	L^3
Velocity	LT^{-1}
Acceleration	LT^{-2}
Volume flowrate	L^3T^{-1}
Mass flowrate	MT^{-1}
Pressure	$ML^{-1}T^{-2}$
Shear stress	$ML^{-1}T^{-2}$
Angular velocity	T^{-1}
Frequency	T^{-1}
Viscosity	$ML^{-1}T^{-1}$
Kinematic viscosity	L^2T^{-1}
Surface tension	MT^{-2}
Force, Thrust	MLT^{-2}
Moment, torque	ML^2T^{-2}
Power	ML^2T^{-3}
Work, energy	ML^2T^{-2}
Density	ML^{-3}
Temperature	θ

Common non-dimensional groups