The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, AUTUMN SEMESTER 2021-2022

THERMOFLUIDS 2

Time allowed 2 hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL questions

Only a calculator from approved list B may be used in this examination.

Basic Models	***Scientific Calculators***	***Graphical Calculators***
Aurora HC133 Casio HS-5D Deli – DL1654 Sharp EL-233	Aurora AX-582 Casio FX83 family Casio FX85 family Casio FX570 family Casio FX 991 family Sharp EL-531 family	Casio FX9750 family

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: Five printed sheets of formulae
Thermodynamic Properties of Fluids & other data (in S.I. units, 5^{th} edition)
Enthalpy-Entropy chart – A3 sized photocopy

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

Answer all questions

1. [4 marks]

For the flow to be incompressible, we need that [1]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$

We first find the partial derivatives [2]:

$$\frac{\partial u}{\partial x}=2xy,\frac{\partial v}{\partial y}=-2xy,$$

The sum of the two derivatives yields 0 and therefore the velocity field is incompressible [1].

2. [4 marks]

Taking the momentum equation, bringing the gravity term to the right-hand side and integrating twice along x, we obtain [1]:

$$v(x)=\frac{-\rho g}{\mu}\frac{x^2}{2}+ax+b.$$

The boundary conditions are: $v(x=0)=v(x=H)=0$, which yield:

$$a=\frac{\rho g}{\mu}\frac{H}{2},b=0,$$

and thus [1]:

$$v(x)=\frac{\rho g}{2\mu}\left(Hx-x^2\right).$$

The maximum speed in the duct is measured at the centre, $x=H/2$ [2]:

$$v_{max}=\frac{\rho g}{2\mu}\left(\frac{H^2}{2}-\frac{H^2}{4}\right)=\frac{\rho g H^2}{8\mu}=\frac{1000\frac{kg}{m^3}\times\left(-9.81\frac{m}{s^2}\right)\times(0.001\,m)^2}{2\times 0.001\frac{kg}{m\,s}}=1.23\frac{m}{s}.$$

3. [4 marks]

$$\theta(x)=\int_0^{\delta}\frac{u}{U_0}\left(1-\frac{u}{U_0}\right)dy=\int_0^{\delta}\frac{y}{\delta}\left(1-\frac{y}{\delta}\right)dy=\frac{1}{6}\delta(x)[2]$$

$$\tau_w(x)=\rho U_0^2\frac{d\theta}{dx}=\rho U_0^2\frac{1}{6}\frac{d\delta}{dx}$$

$$\tau_w(x)=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}=\frac{\mu U_0}{\delta(x)}$$

$$\rho U_0^2\frac{1}{6}\frac{d\delta}{dx}=\frac{\mu U_0}{\delta(x)}\Longrightarrow\delta(x)d\delta=\frac{6\mu}{\rho U_0}dx\Longrightarrow\int_{\delta(x=0)=0}^{\delta(x)}\delta\,d\delta=\int_{x=0}^{x}\frac{6\mu}{\rho U_0}dx$$

Or [2]:

$$\left[\frac{\delta^2}{2}\right]_0^\delta = \frac{\delta^2}{2} = \frac{6\mu}{\rho U_0}x \Longrightarrow \frac{\delta^2}{x^2} = \frac{12}{\Re_x} \Longrightarrow \frac{\delta}{x} \approx \frac{3.46}{\sqrt{\Re_x}}$$

$$\delta = \sqrt{\frac{12\mu}{\rho U_0}x}$$

4. [4 marks]

$$\Re_L = \frac{UL}{\nu} = \frac{80\frac{m}{s} \times 3m}{0.000018\frac{m^2}{s}} = 1.33 \times 10^7 \rightarrow Turbulent\,[1\,mark]$$

Flow is turbulent. For (a), we use Prandtl's formula:

$$C_D = \frac{0.031}{\Re_L^{1/7}} = \frac{0.031}{\left(1.33 \times 10^7\right)^{1/7}} = 0.00298,$$

And [1]:

$$D = \frac{1}{2}C_D \rho U^2 bL = \frac{1}{2} \times 0.00298 \times 1.2\frac{kg}{m^3} \times \left(80\frac{m}{s}\right)^2 \times 0.6\,m \times 3\,m = 20.6\,N.$$

For (b), the drag coefficient is independent of Re and can be calculated with [1]:

$$C_D = \left(1.89 + 1.62\log\frac{L}{\varepsilon}\right)^{-2.5} = \left(1.89 + 1.62\log\frac{3\,m}{0.001\,m}\right)^{-2.5} = 0.00644,$$

And [1]:

$$D = \frac{1}{2}C_D \rho U^2 bL = \frac{1}{2} \times 0.00644 \times 1.2\frac{kg}{m^3} \times \left(80\frac{m}{s}\right)^2 \times 0.6\,m \times 3\,m = 44.5\,N.$$

5. [4 marks]

Reynolds numb … y of the oil is:

There is no dimensional analysis in the January 2023 exam

$$U_o = \frac{4M_o}{\rho_o \pi d_o^2} = \frac{4\left(\frac{2}{60}\frac{kg}{s}\right)}{879\frac{kg}{m^3} \times \pi \times (0.003\,m)^2} = 5.36\frac{m}{s},$$

which gives [1]:

$$\Re = \frac{\rho_o U_o d_o}{\mu_o} = \frac{879\frac{kg}{m^3} \times 5.36\frac{m}{s} \times 0.003\,m}{0.287\frac{kg}{m \times s}} = 49.$$

The velocity of water will be:

$$U_w = \frac{\mu_w}{\rho_w d_w}\Re = \frac{0.001\frac{kg}{m\times s}}{1000\frac{kg}{m^3}\times 0.05\,m}\times 49 = 0.00098\frac{m}{s},$$

which gives a mass flow rate of [2]:

$$M_w = \frac{\rho_w \pi d_w^2 U_w}{4} = 1000\frac{kg}{m^3}\times\pi\times(0.05\,m)^2\times 0.00098\frac{m}{s}\times\frac{1}{4} = 0.0019\frac{kg}{s}.$$

6. [4 marks]

(a) m=n-k=7-3=4 nondimensional groups [1]

(b)

$\Pi = \gamma\rho_l^a U^b D$

There is no dimensional analysis in the January 2023 exam

$^{+c}T^{-2-b} = M^0 L^0 T^0.$

Therefore: a=-1, b=-2, c=-1, and [2]:

$$\Pi = \frac{\gamma}{\rho_l U^2 D} = \frac{1}{We},$$

where the Weber r

There are only 5 fluid mechanics questions and 5 thermodynamics questions in the January 2023 exam

o of inertial to surface tension forces [1].

There is no dimensional analysis in the January 2023 exam

7. 4 marks

On p.10 of the tables:
specific heat capacity at constant pressure, c_{pg} = 2.01 kJ·kg^{-1}K^{-1}. [1]
dynamic viscosity, μ_g = 12.0×10^{-6} kg·m^{-1}s^{-1}. [1]
thermal conductivity, k_g = 24.8×10^{-6} kW·m^{-1}K^{-1}. [1]

Prantdl number is then:

$$Pr=\frac{2.01\times 12.0\times 10^{-6}}{24.8\times 10^{-6}}=0.9726$$

this matches well the value in the tables. Units were not converted to J and W because the kJ and kW cancelled the factor of 1000 on each. [1]

8. 4 marks

Thermal power is the product of mass flow rate and the change in specific enthalpy. Starting enthalpy is 125.7 kJ·kg^{-1} [1] and enthalpy at the end is for superheated steam, have to interpolate from 500 and 600°C:

$$\frac{h_{¿}-h_{500}}{h_{600}-h_{500}}=\frac{550-500}{600-500}$$

$$\frac{h_{¿}-3295}{3573-3295}=\frac{50}{100}$$

therefore h_{out} = 3434 kJ·kg^{-1}. [1]

therefore the thermal power is:

$$\dot{Q}=\dot{m}\Delta h=20\times(3434-125.7)=66{,}166\,kW$$

i.e. 66.166 MW [2].

9. 4 marks

At 0.05 bar, saturation entropy is 8.394 kJ·kg^{-1}K^{-1}, so it is wet steam, made up of s_g = 8.394 kJ·kg^{-1}K^{-1} and s_f = 0.476 kJ·kg^{-1}K^{-1}. Dryness fraction x:

$$(1-x)s_f+x\,s_g=6.394\rightarrow x=0.747$$

h_{out} is therefore partly h_g = 2561 kJ·kg^{-1} and partly h_f = 138 kJ·kg^{-1}:

$$(1-x)h_f+x\,h_g=h\rightarrow h=1949.0$$

useful shortcut is to use the chart to get the same value

[2]

and using the isentropic efficiency:

$$\eta_{isen.turbine}=\frac{actual\,work\,out}{3411.5-1949}=0.96\rightarrow actual\,work\,out=1404.0$$

units kJ·kg^{-1}. [2]

10. 4 marks

At 80% humidity and 40°C, it is possible to find the vapour pressure using the formula, and $p_{g,40C}$ = 0.07375 bar (from tables):

$$\phi=\frac{p_s}{p_g}\rightarrow 0.8=\frac{p_s}{0.07375}\rightarrow p_s=0.059$$

units bar [1]. At 21°C, p_g = 0.02486 bar. Since this is less than 0.059 bar, it means that the air will not support the full amount of vapour in the air, and that some vapour will condense leaving the room at 100% humidity [1]. The absolute humidity will be, by using the formula:

$$\omega = 0.622\frac{p_s}{(p-p_s)} = 0.622\frac{0.02486}{(1-0.02486)} = 0.0159[1]$$

100% humidity is very uncomfortable for people, and this is unacceptable. [1]

11. 4 marks, [1] for each arrow and label:

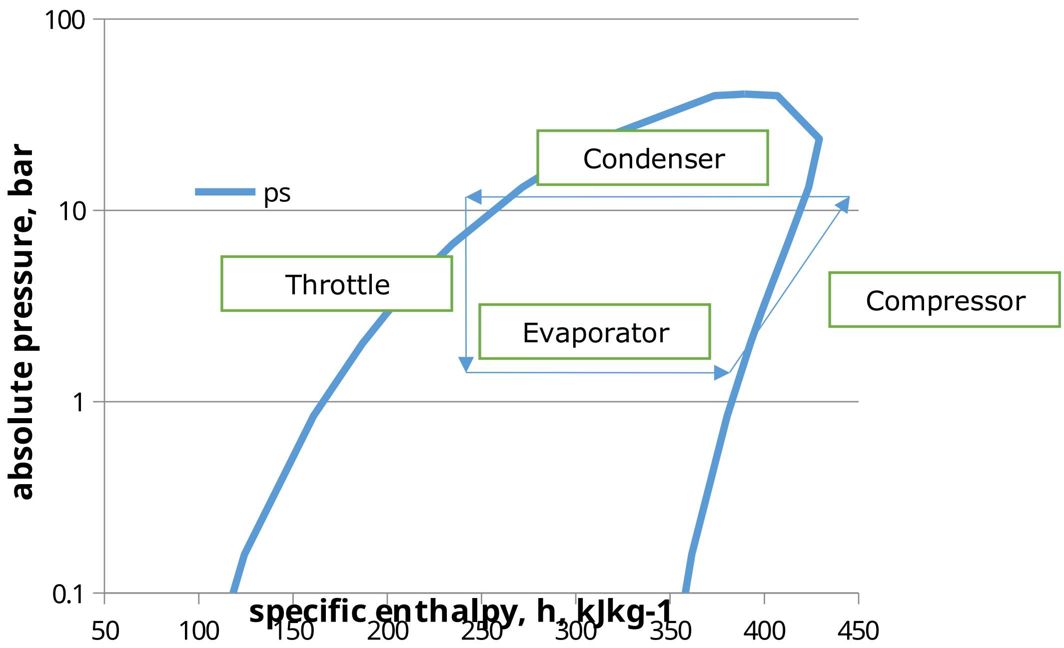

Figure Q10

12. 4 marks

use the formulae to see what is going on:

$$SSC = \frac{3600}{net\ specific\ power\ out} \rightarrow 3\,kg/kWh = \frac{3600\,kJ/kWh}{\dot{w}\ kJ/kg} \rightarrow \dot{w} = 1200\,kJ\cdot kg^{-1}[1]$$

Therefore gross or turbine specific work output is:

$$r_w = \frac{net\,work\,out}{gross\,work\,out} \rightarrow \dot{w}_{gross} = \frac{1200}{0.97} = 1237.1\,kJ\cdot kg^{-1}[1]$$

and the flow rate of water is by the pump power, which is 1237.1-1200.0 = 37.1 $kJ\cdot kg^{-1}$ and the pump power which is 1 MW (1000 kW), therefore:

$$\dot{m} = \frac{1000\,kJ/s}{37.1\,kJ/kg} = 26.95\,kg\cdot s^{-1}[1]$$

and turbine power is the product of mass flow rate and power: 32,344 kW or 32.3 MW [1].

alternative method, small difference due to rounding errors:

$$r_w = 0.97 = \frac{net\,work\,out}{gross\,work\,out} = \frac{\dot{W}_{turbine} - \dot{W}_{pump}}{\dot{W}_{turbine}} \rightarrow \dot{W}_{turbine} = \frac{\dot{W}_{pump}}{0.03} = \frac{1000}{0.03} = 33{,}333\,kW\,[1]$$

net work in kWh/s:

$$\dot{w}_{net} = \frac{\dot{w}_{net}\,kJ/s}{3600\,kJ/kWh} = \frac{32{,}333}{3600} = 8.981\,kWh/s$$

and finally mass flow rate:

$$\dot{m}_{water} = 8.981 \frac{kWh}{s} \times \frac{3\,kg}{kWh} = 26.94\,kg/s\,[1]$$

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