A LEVEL 2 MODULE, SPRING SEMESTER 2020-2021

THERMODYNAMICS AND FLUID MECHANICS 2

SOLUTIONS

Part A: Thermodynamics

1. A solid biomass has an ultimate gravimmetric composition of 48% carbon, 6% hydrogen and 44% oxygen. It is burned completely in air in a power generation plant, and produces an exhaust stream with 5% oxygen in the wet products by volume.

(a) Determine the mass and number of moles of carbon dioxide and water vapour produced by the stoichiometric combustion of 1 kg of the biomass. [4]

$C + O_2 \rightarrow CO_2$
$\tilde{m}_i$: 12 32 44

$\therefore$ for 0.48 kg C, $\frac{44}{12} \times 0.48 = 1.76$ kg [1]

No of Moles of CO_2 per kg fuel

$\frac{1.76 \text{ kg}}{44 \text{ kg/kmol}} = 0.040$ kmol [1]
$= 40$ moles

$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$
$\tilde{m}_i$: 2 16 18

for 0.06 kg H, $\frac{18}{2} \times 0.06 = 0.54$ kg [1]

No of moles of H_2O per kg fuel

$\frac{0.54}{18} = 0.03$ kmol
$= 30$ mols. [1]

in absence of correct form of responses, 1 mark for getting the molar masses

(b) Calculate the mass of air required for stoichiometric combustion of 1 kg of biomass. [5]

First determine amount of O_2 required

Mass of CO_2 − Mass of C = 1.76 − 0.48 = 1.28 kg O_2
mass of H_2O − mass of H = 0.54 − 0.06 = 0.48 kg O_2
mass of O in fuel = 0.44

$\therefore$ Mass of O_2 required is : 1.28 + 0.48 − 0.44 = 1.32 kg [2]

that is per kg of fuel, and assuming the O in fuel is used to form the C & H components.

$\therefore$ Mass of air required, using the ratio of air to O_2:

4.29 kg air/kg O_2 × 1.32 kg O_2 → 5.66 kg air per kg fuel. [3]

(c) Given that the volume fractions of the exhaust gases are N_2 31.9%, O_2 4.8%, CO_2 36.2 %, H_2O 27.1%, use a table of the wet products of combustion to help determine the specific heat capacity at constant pressure for the product gas mixture at 300 K. [4]

Given: $0.05 = \frac{x}{15.5 + 3.76x + 40 + 30}$ → $4.275 + 0.188x = x$ ∴ $x = 5.26$

Table of products:

component	n_i	n_i	n_i/n	$\tilde{m}_i$ (kg/kmol) [1]	c_{p_i} [1]	$\frac{n_i}{n}\tilde{m}_i$ [1]	$\frac{M_i}{M} = \frac{n_i}{n}\frac{\tilde{m}_i}{\tilde{M}}$ [1]	$\times c_{p_i}$
CO_2	40	40	0.362	44	0.846	15.928	0.509	0.431
H_2O	30	30	0.271	18	1.864	4.878	0.156	0.291
N_2	15.5 + 3.76x	35.30	0.319	28	1.040	8.932	0.285	0.296
O_2	x	5.26	0.048	32	0.918	1.536	0.049	0.045
	GIVEN		Given			$\tilde{m} = \Sigma = 31.274$ kg/kmol.	$c_p =$	1.063 kJ/kgK

specific molar is acceptable using volume fractions, works out as 33 kJ/kmolK

2. (a) A single pass, counterflow shell and tube heat exchanger uses tubes of diameter 20 mm with water flowing at the rate of 0.1 kg/s in each tube at 20 °C. Calculate the heat transfer coefficient at the inner wall of the tube given the Nusselt number correlations:

$$Nu = 0.023Re^{0.8}Pr^{0.4} \text{ For turbulent flow, and}$$

$$Nu = 3.66 \text{ For laminar flow.}$$

[5]

[2] Tables: $\rho_{H_2O} = \frac{1}{0.0010018} = 998 \text{ kg/m}^3$ $\quad u = \frac{\dot{m}}{\rho A} = \frac{0.1}{998 \times \pi/4 \times 0.02^2} = 0.319 \text{ m/s}$

$\mu_{H_2O} = 1002 \times 10^{-6}$ kg/ms

$Re = \frac{\rho u D}{\mu} = \frac{998 \times 0.319 \times 0.02}{1002 \times 10^{-6}} = 6353$

[1] ∴ turbulent & $Nu = 0.023 \times 6353^{0.8} \times 6.95^{0.4} = 55$

[2] $Nu = \frac{hd}{k} \Rightarrow 55 = \frac{h \times 0.02}{603 \times 10^{-6} \times 10^3}$ → $h = 1658$ W/m²K

Tables

half mark deducted on Re for using air or steam rather than water – full marks if method ok following

(c) The heat exchanger has a shell and tube arrangement, containing 10 tubes, with a flow of air entering the shell at 1000 K. The air enters at a flow rate of 0.5 kg/s and the heat transfer coefficient on the outer surface of the tubes is 100 W/m^{2}K, and the conduction resistance of the pipe wall is negligible. Calculate the overall heat transfer coefficient and the length of the tubes in order to produce a gas exit temperature of 400 K. The correction factor may be assumed to be 1.0. [8]

at least half marks for method if wrong values used

$$\dot{Q} = \dot{m}_{air} C_{p\,air} \Delta T_{air}$$

Table

$$\bar{C}_{p\,air} = \frac{C_{p,800°C} + C_{p,200°C}}{2} = \frac{1.1411 + 1.0135}{2} = 1.0773 \text{ kJ/kg K} \quad [1]$$

$$\dot{Q} = 0.5 \times 1.0773 \times (800-200) = 323 \text{ kW} \quad [1]$$

$$U = \frac{1}{R_{TH} A} \qquad R_{TH} = \frac{1}{h_{out} A_{out}} + \frac{1}{h_{in} A_{in}}$$

$$U = \frac{1}{\frac{1}{100} + \frac{1}{1658}} = \underline{94 \text{ W/m}^2\text{K}} \quad [1]$$

$$\Delta T_m = \frac{\Delta T_A - \Delta T_B}{\ln \Delta T_A / \Delta T_B}$$

$$\Delta T_W = \frac{323}{4.2 \times 1} = 76 \text{ °C} \quad [2]$$

$$\Delta T_M = \frac{(1273-96)-(673-20)}{\ln\left(\frac{1273-96}{673-20}\right)} = 889 \text{ °C} \quad [2]$$

$$Q = UA\Delta T_M$$

$$\therefore 323000 = 94 \times (\pi \times 0.02 \times L \times 10) \times 889 \quad [1]$$

$$\rightarrow \underline{L = 6.13 \text{ m}}$$

Part B: Fluid Mechanics

3. A small airship, shown in Figure Q.1a, is cruising at a steady speed and constant altitude at a velocity of 20 km·h^{-1}. It is filled entirely with 4000 m^3 of helium. The temperature of the surrounding air is 286 K.

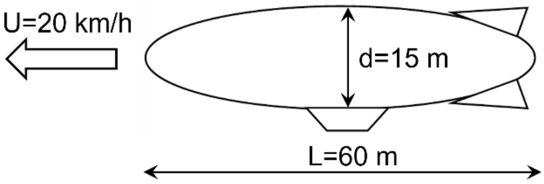


Figure Q.1a: Airship dimensions. The airship cross-section is circular.

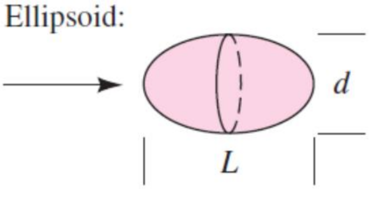
(a) Calculate the Reynolds number of the airship based on diameter. Given the transition to turbulent flow occurs at Re = 2×10^5, state whether the flow past the airship is turbulent or laminar. (Take the density of air to be 1.2 kg·m^{-3} and the kinematic viscosity to be 1.4×10^{-5} m^2·s^{-1}). [2]

$$Re = \frac{Ud}{\nu} = \frac{5.55\ \frac{m}{s} \times 15\ m}{1.4 \times 10^{-5}\ \frac{m^2}{s}} = 5.94 \times 10^6$$

Therefore flow is turbulent. Marking scheme: [-1] for any mistake.

(b) The airship can be approximated by a three-dimensional ellipsoid. Using the information in Table Q.1b that shows C_D values as a function of L/d, estimate the drag coefficient of the airship, and calculate the drag force. [5]

Ellipsoid:



L/d	Laminar	Turbulent
0.75	0.5	0.2
1	0.47	0.2
2	0.27	0.13
4	0.25	0.1
8	0.2	0.08

Figure Q.1b: Drag coefficients of an ellipsoid, based on frontal area.

L/d=4 and thus C_D=0.1 [2]. Frontal area: A=pi*D^2/4=176.6 m^2 [2]. Drag force [1].

$$D = \frac{1}{2} C_D \rho U^2 A = \frac{1}{2} \times 0.1 \times 1.2 \frac{kg}{m^3} \times \left(5.55\ \frac{m}{s}\right)^2 \times 176.6\ m^2 = 326.4\ N$$

(c) A problem develops with the airship and helium leaks through a nozzle to atmospheric pressure of 0.99 bar. By treating the flow through the system as one-dimensional isentropic, and neglecting all friction losses, calculate the Mach number of the helium at the nozzle if the stagnation pressure of helium in the airship is 1.2 bar. Take the ratio of specific heats for Helium (g) to be 1.66 and the gas constant for helium, R_{helium} to be 2077 $J \cdot kg^{-1} \cdot K^{-1}$ [4]

$$\frac{p_0}{p} = \left[1 + Ma^2 \frac{\gamma - 1}{2}\right]^{\frac{\gamma}{\gamma-1}} \Longrightarrow Ma^2 = \left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma - 1} =$$

$$\left[= \left(\frac{1.2\ bar}{0.99\ bar}\right)^{\frac{1.66-1}{1.66}} - 1\right]\frac{2}{1.66 - 1} = 0.241$$

Therefore, Ma=sqrt(0.241)=0.491 [3]

(d) Assuming the helium inside the airship has the same temperature as the surrounding air, what is the temperature of helium at the point of exit through the nozzle?

$$\frac{T_0}{T} = 1 + Ma^2 \frac{\gamma - 1}{2} = 1 + 0.491^2 \frac{1.66 - 1}{2} = 1.08$$

Therefore, since T_0=286 K, T=T_0/1.08=265.8 K [3]

4. A centrifugal pump is used to mobilise Q=10 $m^3 \cdot h^{-1}$ of viscous oil, density r=900 $kg \cdot m^{-3}$. The pump being used is the 4.75" pump from the HTO-80 series from MP Pumps Inc, the performance curves for the pump are shown in Figure Q.2.

(a) Using the graph and appropriate calculations, determine the head delivered by the pump, the differential pressure generated, the efficiency of the pump, the power delivered to the fluid and the brake horsepower in Watts, with the pump operating at 3450 rpm. Note that 1 hp=746 W. [5]

From the graph in Fig. Q2, the 4.75" pump at a flow rate of $Q = 10\,m^3/h$ generates a head of $H = 21\,m$ [1]. The differential pressure generated is [1]:

$$\Delta p = \rho g H = 900\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 21\,m = 185.4\,kPa$$

From the graph, the pump efficiency is about $\eta = 0.51$ [1]. The power delivered to the fluid is [1]:

$$P_w = \rho g Q H = 900\frac{kg}{m^3} * 9.81\frac{m}{s^2} * 0.0028\,\frac{m^3}{s} * 21\,m = 519\,W$$

The pump input power [1] can either be calculated as $P = P_w/\,\eta = 1018\,W$, or extracted from the graph obtaining $P = 1.3\,hp = 970\,W$. Is it also possible to extract the brake horsepower from the graph and then use the water horsepower evaluated above to calculate the efficiency, this would give a similar value $\eta = \frac{P_w}{P} = 0.535$.

(b) If the eye of the pump has a diameter of D=5 cm, what should be the lowest oil pressure at the inlet to prevent cavitation? Consider that the vapour pressure of oil at the inlet is p_v=15 kPa. [5]

At $Q = 10\,m^3/h$, the NPSH of the pump is about NPSH=3 m [1]. To avoid cavitation, it is necessary that [1]:

$$\frac{p_i}{\rho g} + \frac{v_i^2}{2g} - \frac{p_v}{\rho g} > NPSH \Longrightarrow p_i \geq \rho g \left(NPSH - \frac{v_i^2}{2g} + \frac{p_v}{\rho g} \right)$$

The velocity of the water at the inlet is [2]:

$$v_i = \frac{4Q}{\pi D_i^2} = \frac{4 * 0.0028\,\frac{m^3}{s}}{\pi * (0.05\,m)^2} = 1.43\,\frac{m}{s}$$

So that [1]:

$$p_i \geq 900\frac{kg}{m^3} * 9.81\frac{m}{s^2} * \left[3\,m - \frac{\left(1.43\frac{m}{s}\right)^2}{2 * 9.81\frac{m}{s^2}} + \frac{15000\,Pa}{900\frac{kg}{m^3} * 9.81\frac{m}{s^2}} \right] = 40525\,Pa$$

(c) The company wish to operate at a higher flowrate of Q=19 $m^3 \cdot h^{-1}$. The engineer considers buying a larger pump from the same series and operate at the same speed. Using pump similarity laws, work out what size pump should be bought and what power it will require. [3]

Using similarity rules [2]:

$$\frac{Q_2}{Q_1} = \left(\frac{D_2}{D_1}\right)^3 \Longrightarrow D_2 = D_1 \left(\frac{Q_2}{Q_1}\right)^{\frac{1}{3}} = 4.75'' \times \left(\frac{19\, m^3/h}{10\, m^3/h}\right)^{\frac{1}{3}} = 5.9''$$

The power it will require [1]:

$$P_2 = P_1 \left(\frac{D_2}{D_1}\right)^5 = 1018\, W \times \left(\frac{5.9''}{4.75''}\right)^5 = 3008\, W$$

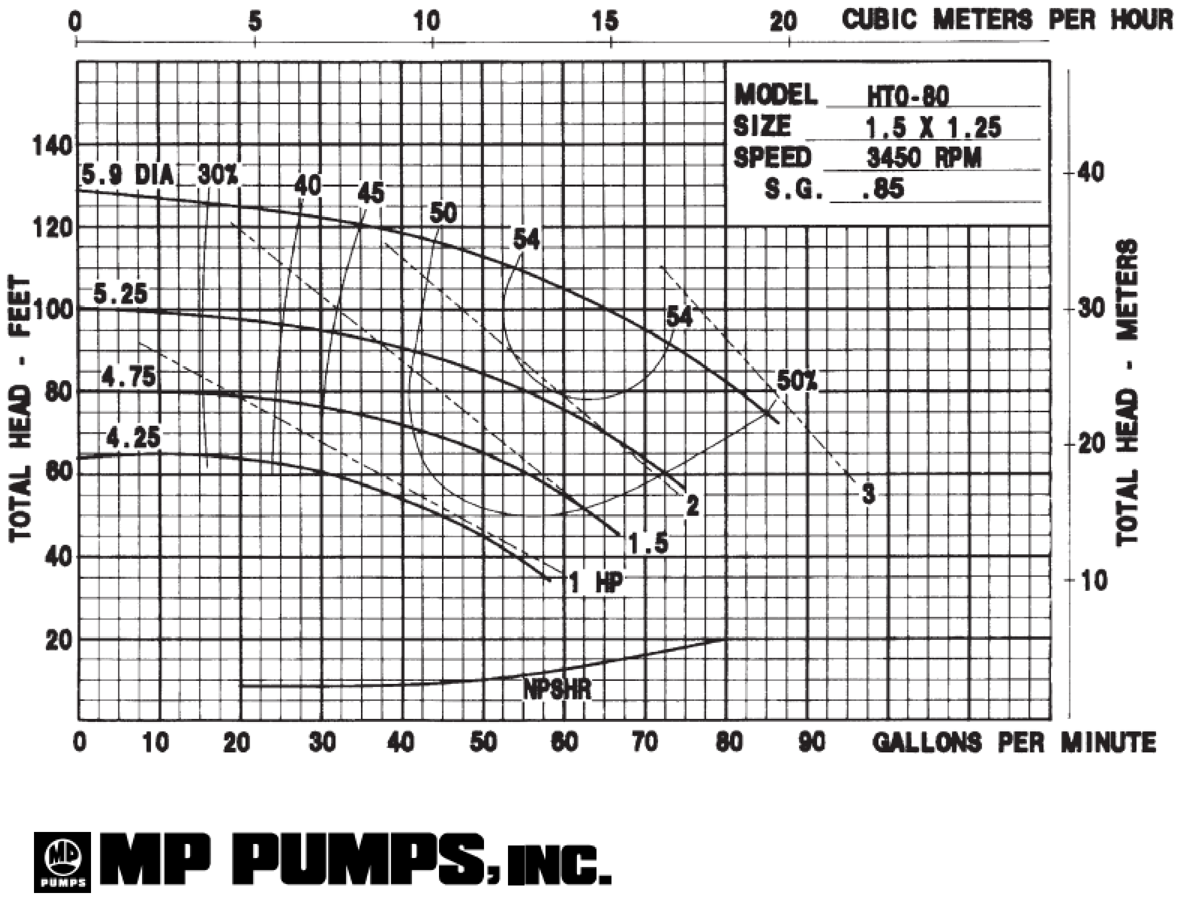


Figure Q.2: Performance curves for pump HTO-80.