Feeling the pressure.

December 2020

1. A man blows up a party balloon, compressing the air to 0.1 barg from 1 atm and 15°C. The polytropic index of the compression process is 1.4. What is the temperature of the compressed air?
2. What work is done if the balloon in qu.1 is 20 cm in diameter?
3. If the air had been compressed in a closed system process, say in a cylinder, what would the work have been? What is the reason for the difference between the closed and open process work?
4. A reciprocating single stage air compressor delivers air at 7 barg. The atmospheric conditions are 1 bar abs and 15°C. The piston has 50 mm diameter and a stroke of 50 mm. The clearance volume is 5% of the swept volume. The polytropic index of the compression process is 1.3. What is the volumetric efficiency?
5. The compressor works at a speed of 480 cycles per minute. What is the required power to compress the air?
6. What would be the isothermal work for the same compression, and hence what is the isothermal efficiency?
7. What is the heat transfer to the cylinder jacket in order to maintain this process?
8. A two-stage compressor is used to compress atmospheric air at 1 atm and 15°C to 20 barg. This is done with minimum work input. What is the intermediate pressure?
9. Show that for this benefit to be achieved, complete intercooling must be applied. What is the effect on the second stage work if intercooling is not applied?

1. Use the formula for polytropic compression, $pv^n = c$, where c is a constant. This formula is required in the form of p and T rather than v. Convert as follows:

$$pv^n = c;\ pv = mRT;\ v = {}^{mRT}\!/_{p}$$

$$p_1\left({}^{mRT_1}\!/_{p_1}\right)^n = p_2\left({}^{mRT_2}\!/_{p_2}\right)^n$$

$$\frac{T_1^n}{p_1^{n-1}} = \frac{T_2^n}{p_2^{n-1}};\ \frac{p_2^{\frac{n-1}{n}}}{p_1^{\frac{n-1}{n}}} = \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Therefore, $T_2 = (1.11325/1.01325)^{0.4/1.4} \times 288 = 295.8$ K, or 22.3°C.

2. Work done is for a steady flow process, since it is an open system. Therefore,

$$w = -\int_1^2 v.dp;\ pv^n = c;\ v = \frac{c^{1/n}}{p^{1/n}}$$

$$w = -\int_1^2 \frac{c^{1/n}}{p^{1/n}}.dp = -c^{1/n}\left[\frac{p^{1-1/n}}{1-1/n}\right]_1^2 = -c^{1/n}\frac{n}{n-1}\left[p_2^{\frac{n-1}{n}} - p_1^{\frac{n-1}{n}}\right]$$

$$w = -\left(pv^n\right)^{1/n}\frac{n}{n-1}\left[p_2^{\frac{n-1}{n}} - p_1^{\frac{n-1}{n}}\right] = -\frac{n}{n-1}\left[p_2^{\frac{n-1}{n}}\left(p_2v_2^n\right)^{1/n} - p_1^{\frac{n-1}{n}}\left(p_1v_1^n\right)^{1/n}\right]$$

$$w = \frac{n}{1-n}\left[p_2^{\frac{n-1}{n}+\frac{1}{n}}v_2 - p_2^{\frac{n-1}{n}+\frac{1}{n}}v_2\right] = \frac{n}{1-n}\left[p_2v_2 - p_1v_1\right]$$

That is per kg of air. The mass of air in the balloon can be found from the gas law, pV=mRT (note shift to capital for total volume). $m = 111325\times(\pi\times0.2^3/6)/(287\times295.8) = 0.0055$ kg, or 5.5 g. Therefore work is

$$w = m\frac{n}{1-n}\left[p_2v_2 - p_1v_1\right] = m\frac{n}{1-n}R\left[T_2 - T_1\right] = 0.0055\frac{1.4}{0.4}287\left[295.8 - 288\right] = 43\text{J}.$$

3. By similar process of deduction for the closed process starting from $w = -\int_1^2 p.dv$ the work in the closed system is $w = m\frac{1}{n-1}R\left[T_2 - T_1\right]$. The work is therefore 30.8 J. The difference is 12.2 J. The difference is the flow work, $m(p_2v_2 - p_1v_1)$ from the difference between u and h (since h = u + pv) in the First Law, q + w = Δu and q + w = Δh. Prove this since flow work is $mR(T_2 - T_1) = 12.3$ J. This is the energy stored in the gas as springy energy, which is contained by rigid walls in the case of the closed process, but is contained by the gas itself in the open process in order to maintain a flow.

4. $\eta_{vol} = \frac{V_i}{V_s}$, and V_i in this case is worked out from the expansion of the clearance volume thus:

$$V_s = \frac{\pi d^2 L}{4} = \frac{\pi\times0.05^3}{4} = 9.81\times10^{-5}\ \text{m}^3.$$

$$V_c = 0.05V_s = 4.9\times10^{-6}$$

Expand the clearance volume across the pressure difference to get the wasted stroke:

$$p_{low} V_{exp}^{n} = p_{high} V_{c}^{n}$$

$$V_{exp} = \sqrt[1.3]{8 \times \left(4.9 \times 10^{-6}\right)^{1.3}} = 2.43 \times 10^{-5} \text{ m}^3.$$

$V_i = V_c + V_s - V_{exp}$

$V_i = 7.87 \times 10^{-5}$ m^3 and hence $\eta_{vol} = 0.80$ or 80%.

5. $W_{act} = \frac{\dot{m}n}{n-1} R\left(T_2 - T_1\right)$ need mass flow rate, from

$$\dot{m} = \eta_{vol} V_s \rho N = 0.8 \times 9.81 \times 10^{-5} \times \frac{100000}{287 \times 288} \times \frac{480}{60} = 0.00076 \text{kg/s}$$

Also require the temperature at the higher pressure from:

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$$

$$T_{high} = 288 \times \left(\frac{8}{1}\right)^{\frac{1.3-1}{1.3}} = 465\text{K}$$

The work is then 167.3 W.

6. $W_{iso} = \dot{m}RT_1 \ln\frac{p_2}{p_1} = 0.00076 \times 287 \times 288 \times \ln\frac{8}{1} = 130\text{W}$ Isothermal efficiency is then 130/167 = 0.78 or 78%.

7. $Q = \dot{m}\frac{\gamma - n}{1-n} c_v (T_2 - T_1)$

c_v is not constant since the temperature changes during the work done. A good compromise is to take the value at the average temperature, which is (288+465)/2 = 377 K. Tables show that at this temperature c_v = 724 J/kgK. Putting this in together with the other known values:

$$Q = 7.6 \times 10^{-4} \times \frac{1.4 - 1.3}{1 - 1.3} \times 724 \times (465 - 288) = 32.5W$$

8. $p_i = \sqrt{21 \times 1.01325} = 4.61$ bar abs.

9. Since $W_{act} = \frac{\dot{m}n}{n-1} R\left(T_2 - T_1\right)$ and $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$, if T_1 is not the same in both the stages, then the difference between them will not be the same (the pressure relationship is not linear). Hence we know that minimum work depends on the work in each stage being the same, complete inter-cooling must be applied.