

### Convection heat transfer calculation examples:

1. The convective heat transfer to a plane wall inner surface is  $9 \text{ W/m}^2\text{K}$ , and on the outer wall it is  $95 \text{ W/m}^2\text{K}$ . The wall is 2.3m high and 3.1m wide. Ignoring the conduction of the wall find the thermal resistance for the inside and outside convective conditions, and hence for the overall wall. [0.0171 K/W]
2. What is the heat transfer in question 1 if the inner air is at  $18^\circ\text{C}$  and the outer air is at  $1^\circ\text{C}$ ? Sketch the temperature profile through the wall. [994 W]
3. A high pressure hot water pipe of inner diameter 28mm carries water at  $347^\circ\text{C}$ , 160 bar. The inner wall convective heat transfer coefficient is *assumed* to be  $940 \text{ W/m}^2\text{K}$ . The pipe is made of stainless steel ( $k=15\text{W/mK}$ ) and has a wall thickness of 6mm. It is insulated by 100mm of refractory material having  $k=0.8\text{W/mK}$ . The surrounding air temperature is  $25^\circ\text{C}$  and the heat transfer coefficient is  $10\text{W/m}^2\text{K}$ . What is the heat loss per m length of pipe? What is the external surface temperature? [638 W,  $109^\circ\text{C}$ ]
4. The pipe in Q3 is part of a heat exchanger and it is useful to know the overall heat transfer coefficient for this insulated part of it to compare with the uninsulated part in the heat exchanger. Starting with the thermal resistance in Q3, calculate the overall heat transfer coefficient based on 1 m length of the inner diameter of the pipe to produce the overall heat transfer coefficient, and compare it to the overall heat transfer coefficient in the heat exchanger, which instead of insulation has a convective heat transfer coefficient of  $90 \text{ W/m}^2\text{K}$ .
5. Calculate convective heat transfer coefficient for the inner surface in question 3 using the Colburn correlation ( $Nu_d=0.023Re_d^{0.8}Pr^{0.4}$ ). Given the velocity of water in the pipe is 3m/s, the density is  $613\text{kg/m}^3$ , the viscosity is  $9\times 10^{-5}\text{kg/ms}$ , the Prandtl number is 0.94 and the conductivity is  $0.54\text{W/mK}$ . What effect will this have on the calculation of heat loss in question 3? [17,466  $\text{W/m}^2\text{K}$ ]
6. Calculate the Grashof number for a vertical surface 0.5m high and 1m wide, in a room of still air, if the air temperature is  $18^\circ\text{C}$  and the surface temperature is  $150^\circ\text{C}$ . Use the Nusselt correlation:  
$$Nu_x = 0.59 (Gr_L Pr)^{0.25} \text{ if } 10^3 < GrPr < 10^9 \text{ and } Nu_x = 0.13 (Gr_L Pr)^{0.33} \text{ if } 10^9 < GrPr < 10^{12}.$$
 Use  $\beta=1/T_f$ , where  $T_f$  is the film temperature (the mean value of the surface and the surrounding gas), and take the properties of air from the tables of dry air at low pressure at  $T_f$ . What is the heat transfer from the surface to the room by convection? [403 W]

### Solutions convection heat transfer:

1. Convective thermal resistance is  $1/hA$ , inside  $1/9 \times 2.3 \times 3.1 = 0.0156 \text{K/W}$  and outside  $1/95 \times 2.3 \times 3.1 = 0.0015 \text{K/W}$ . Total thermal resistance is now  $0.0171 \text{K/W}$ . The external resistance is much less, indicating that it transfers heat well into the wall because of the enhanced heat transfer coefficient.
2. For these temperatures, the heat transfer is  $q = \Delta T/R_{th} = 17/0.0171 = 994 \text{W}$ .
3. Use the formula for thermal resistance of axisymmetric pipe wall and insulation and the convection at the inner and outer surfaces to produce:

$$\dot{q}' = \frac{-(347 - 25)}{\frac{1}{940 \cdot 2\pi \cdot 0.014} + \frac{\ln(0.020/0.014)}{2\pi \cdot 15} + \frac{\ln(0.12/0.020)}{2\pi \cdot 0.8} + \frac{1}{10 \cdot 2\pi \cdot 0.12}}$$

and the heat transfer is  $322/(0.012 + 0.0038 + 0.357 + 0.132) = 322/0.505 = 638 \text{W/m}$ . External temperature is given by the convective calculation on the exterior only i.e.  $q = \Delta T/R_{th}$ , so  $638 = (T_{air} - T_s)/0.132$  so  $T_s = 109^\circ\text{C}$ .

4. Overall heat transfer coefficient is the thermal resistance for  $1 \text{ m}^2$  of the heat exchange surface. The previous question has the thermal resistance for a  $1 \text{ m}$  length of the pipe. For the thermal resistance in this case based on the area of the inner diameter of the pipe and  $1 \text{ m}$  length so that it is consistent with the  $R_{th}$  calculation in Q3:

$$UA = U(2\pi \cdot 0.014) = \frac{1}{\frac{1}{940 \cdot 2\pi \cdot 0.014} + \frac{\ln(0.020/0.014)}{2\pi \cdot 15} + \frac{\ln(0.12/0.020)}{2\pi \cdot 0.8} + \frac{1}{10 \cdot 2\pi \cdot 0.12}}$$

$$U = 22.5 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$$

For the case with no insulation and convection heat transfer coefficient of  $90 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$  on the outer surface of the pipe:

$$U(2\pi \cdot 0.014) = \frac{1}{\frac{1}{940 \cdot 2\pi \cdot 0.014} + \frac{\ln\left(\frac{0.020}{0.014}\right)}{2\pi \cdot 15} + \frac{1}{90 \times 2\pi \cdot 0.02}} = 1/(0.012 + 0.0038 + 0.0884)$$

$$U = 109 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$$

5. Inner surface  $Nu_d = 0.023 Re_d^{0.8} Pr^{0.4}$  where  $Re = \rho U D / \mu$ ,  $Re = 613 \times 3 \times 0.028 / 9 \times 10^{-5} = 572,133$ .  $Nu_d = 0.023 \times 572,133^{0.8} \times 0.94^{0.4} = 905$ . Knowing that  $h = Nu \cdot k / D$ ,  $h = 17,466 \text{ W/m}^2 \cdot \text{K}$ . The effect of this will be to decrease the convective thermal resistance. So  $1/17466 \cdot 2\pi \cdot 0.014$ , making it effectively zero. The thermal resistance is decreased by  $0.012$  out of  $0.505$  or  $2.4\%$ , and hence there will be  $2.4\%$  more heat transfer or  $653 \text{ W/m}$ . Therefore a small effect.
6. Film temperature is average of wall and fluid temp =  $(291 + 423)/2 = 357 \text{K}$ , take properties at  $350 \text{K}$

to reasonable accuracy.  $Gr = \frac{g \beta^3 \rho^2 \Delta T}{\mu^2}$  and use  $\nu = \mu / \rho$ , so  $Gr =$

$$9.81 \cdot 1 / ((291 + 423)/2) \cdot 0.5^3 \cdot (150 - 18) / (2.056 \cdot 10^{-5})^2 = 1\,072\,601\,784 \text{ or } 1.073 \cdot 10^9$$

Work out the product of  $GrPr = 1.073 \cdot 10^9 \times 0.697 = 7.48 \times 10^8$ , where  $Pr$  is from p.16 at  $350 \text{K}$ .

Therefore the first  $Nu$  correlation is appropriate:  $Nu = 0.59(GrPr)^{0.25}$

$$Nu = 0.59(7.48 \times 10^8)^{0.25} = 97.6$$

$h = Nu \cdot k / D = 97.6 \cdot 0.03003 / 0.5 = 5.86 \text{ W/m}^2 \cdot \text{K}$ , where  $k$  is from p.16 at  $350 \text{K}$ . Therefore heat transfer rate is:

$$Q = hA(T_s - T_{air}) = 5.86 \cdot 0.5 \cdot 1 \cdot (150 - 18) = 386.9 \text{ W}$$