COMPRESSIBLE FLOW

SEMINAR 8 EXAMPLES

1. A certain aircraft flies at the same Mach number regardless of its altitude. Compared to its speed at 12000 m (ISA conditions), it flies 127 km/h faster at sea level. Determine its Mach number.

Ans: 0.78

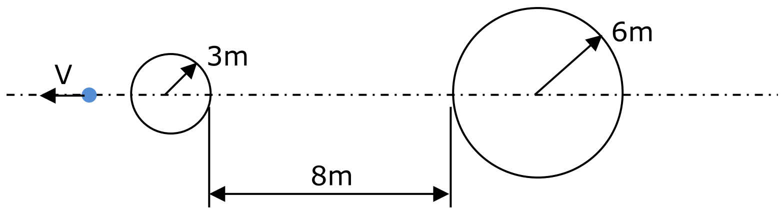
2. An ideal gas flows adiabatically through a duct. At section 1 p_1 = 140 kPa , T_1 = 260°C and V_1 = 75 m/s. Downstream at section 2 p_2 = 30 kPa , T_2 = 207°C. Calculate V_2 and the change in entropy $s_2 - s_1$ if the gas is a) air with γ=1.4and b) Argon with γ=1.67.

Ans: a) 335 m/s, 337 J/KgK; b) 246 m/s, 266 J/KgK

3. An aeroplane flies at Mach 0.8 in air at 15°C and 100 kPa . Calculate the stagnation temperature and pressure.

Ans: 325 K, 152.4 kPa

4. A particle is moving supersonically in air at 1.01325 bar and 288.15K. From the two disturbance spheres shown compute: a) Mach angle, b) particle Mach number, and c) particle velocity.



Ans: 10.2°, 5.7, 1940 m/s

5. (extension question) An air tank of volume 1.5 m^3 is at 800 kPa and 20°C when it begins exhausting through a converging nozzle to sea-level conditions. The throat area is 0.75 cm^2. Estimate a) the initial mass flow; b) the time to blow down to 500 kPa (hint: recall that $\dot{m} = \frac{dm}{dt}$) and c) the time when the nozzle ceases being choked.

Ans: 0.142 kg/s, 47.3s, 143.6s

Solutions

1. From the international standard atmosphere page in the steam tables, at sea level air temperature is 288.15K and at 12,000m air temperature is 216.7K. From the "properties of dry air" page in the steam tables, at 288.15K γ is 1.400 and at 216.7K γ is 1.401[1].

Speed of sound at ground level: $a_g = \sqrt{\gamma_g R T_g} = \sqrt{1.4 \times 287.1 \times 288.15} = 340.3\ m/s$

$$Ma_g = \frac{V_g}{a_g} = \frac{V_g}{340.3}$$

Speed of sound at altitude: $a_a = \sqrt{\gamma_a R T_a} = \sqrt{1.401 \times 287.1 \times 216.7} = 295.2\ m/s$

$$Ma_a = \frac{V_a}{a_a} = \frac{V_a}{295.2}$$

We know that $V_g - V_a = 127 \times \frac{1000}{3600} = 35.28$

Also $Ma_g = Ma_a$

$$Ma(340.3 - 295.2) = 35.28 \text{ so } Ma = \frac{35.28}{45.1} = 0.78$$

2. For adiabatic flow h_o is constant and so

$$h_1 + \tfrac{1}{2}{v_1}^2 = h_o = h_2 + \tfrac{1}{2}{v_2}^2$$

for a perfect gas $\Delta h = C_p \Delta T$ and thus:

$$h_1 - h_2 = C_p(T_1 - T_2) = \tfrac{1}{2}(v_2^2 - v_1^2)$$

a) For air take C_p =1005 J/kgK (valid for approximate calculations with air), thus:

$$1005(260 - 207) = \tfrac{1}{2}(v_2^2 - 75^2)$$

$$v_2 = 335 m/s$$

From MM1TF1 recall that for a perfect gas with constant C_p:

$$s_2 - s_1 = C_p ln\left(\frac{T_2}{T_1}\right) - Rln\left(\frac{p_2}{p_1}\right) = 1005ln\left(\frac{207 + 273.15}{260 + 273.15}\right) - 287.1ln\left(\frac{30}{140}\right) = 337\ J/kgK$$

b) For Argon, M=39.948 kg/kmol so $R = \frac{8314.4}{39.948} = 208.1\ J/kgK$

For a perfect gas, $C_p = R\frac{\gamma}{\gamma-1}$ so $C_p = 208.1\frac{1.67}{1.67-1} = 518.7\ J/kgK$

As before: $C_p(T_1 - T_2) = \tfrac{1}{2}(v_2^2 - v_1^2)$

$$518.7(260 - 207) = \tfrac{1}{2}(v_2^2 - 75^2)$$

[1] It would have been fine to use γ=1.400 in both cases, the approximation is small.

$$v_2 = 246\ m/s$$

And

$$s_2 - s_1 = C_p ln\left(\frac{T_2}{T_1}\right) - Rln\left(\frac{p_2}{p_1}\right) = 518.7ln\left(\frac{207+273.15}{260+273.15}\right) - 208.1ln\left(\frac{30}{140}\right) = 266\ J/kgK$$

3. For a perfect gas

$$\frac{T_o}{T} = 1 + \frac{v^2}{a^2}\left(\frac{\gamma-1}{2}\right) = 1 + Ma^2\left(\frac{\gamma-1}{2}\right)$$

For air, γ=1.4 hence:

$$\frac{T_o}{(15+273.15)} = 1 + 0.8^2\left(\frac{1.4-1}{2}\right)$$

$$T_o = 325K$$

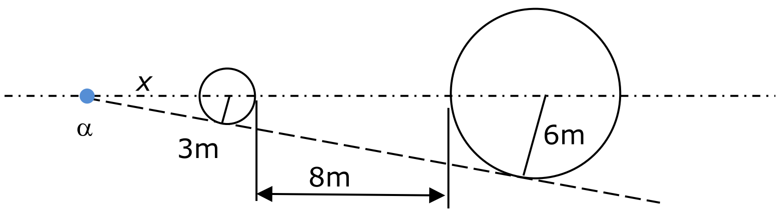
$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left\{1 + Ma^2\left(\frac{\gamma-1}{2}\right)\right\}^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_o}{100000} = \left(\frac{325}{288.15T}\right)^{\frac{1.4}{0.4}} = 152.4\ kPa$$

4. First calculate the speed of sound at this location using γ=1.4, T=288.15K and R=287.1 J/kgK:

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287.1 \times 288.15} = 340.3m/s$$

Using basic geometry, calculate cone angle:



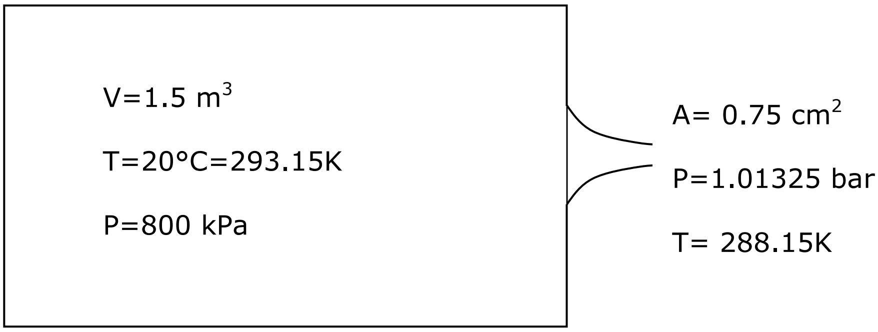
$$\sin\alpha = \frac{3}{x} = \frac{6}{x+3\ +8+6} \implies x + 17 = 2x \implies x = 17$$

$$\therefore\ \sin\alpha = \frac{3}{17} \implies \alpha = 10.2°$$

$$Ma = \frac{V}{a} = \frac{1}{\sin\alpha}\ \therefore Ma = 5.7$$

and $V = a\ Ma = 340.3 \times 5.7 = 1940m/s$

5.



Flow inside tank is stagnant.

For isentropic flow of a perfect gas:

$$\frac{p_o}{p} = \left\{1 + Ma^2\left(\frac{\gamma - 1}{2}\right)\right\}^{\frac{\gamma}{\gamma-1}}$$

The flow is choked when it is sonic in the nozzle throat. Hence Ma>1 for choked flow.

Rearranging, for Ma>1 and γ=1.4, we need:

$$\frac{p_o}{p} = \{1 + Ma^2(0.2)\}^{3.5}$$

$$\frac{\frac{p_o}{p}^{0.286} - 1}{0.2} > 1 \implies \frac{p_o}{p} > 1.893$$

Therefore flow will be choked while pressure in tank (p_o) is greater than 1.918 bar. Starting pressure is 8 bar, hence initial flow will be choked.

For choked conditions mass flow is given by:

$$\dot{m}_{choked} = \sqrt{\gamma}\left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} \rho_o A^* \sqrt{RT_O}$$

A*=0.75x10^{-4}m^2, T_o=293.15K, R=287.1 J/kgK,

$$\dot{m}_{choked} = \sqrt{\gamma}\left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} \rho_o A^* \sqrt{RT_O} = 0.685\frac{p_o}{RT_o}A^*\sqrt{RT_O} = 0.685\frac{p_o A^*}{\sqrt{RT_O}} = 0.142kg/s$$

b) Mass of air in tank is given by $pV = mRT$ where p is the current stagnation pressure, V is the tank volume, R is the gas constant for air and T is the tank temperature. Assume T remains constant. Thus:

$$m = \frac{pV}{RT}$$

Differentiating both sides wrt t gives:

$$\frac{dm}{dt}_{tank} = \frac{V}{RT}\frac{dp}{dt}$$

Mass flowrate through nozzle is $\dot{m} = \frac{0.685A^*}{\sqrt{RT}}p$ (because choked)

$$\frac{dm}{dt}_{tank} = -\dot{m}$$

Because a positive mass flowrate through the nozzle reduces the mass in the tank. Thus:

$$\frac{0.685A^*}{\sqrt{RT}}p = -\frac{V}{RT}\frac{dp}{dt}$$

Rearranging:

$$-\frac{0.685A^*\sqrt{RT}}{V}dt = \frac{dp}{p}$$

Integrate:

$$-\int_{t_{start}}^{t}\frac{0.685A^*\sqrt{RT}}{V}dt = \int_{p_{start}}^{p}\frac{dp}{p}$$

so

$$-\frac{0.685A^*\sqrt{RT}}{V}t = [ln\, p]_{p_{start}}^{p} = ln\left(\frac{p}{p_{start}}\right)$$

(Raise both sides to the power of e: $e^{-\frac{0.685A^*\sqrt{RT}}{V}t} = \frac{p}{p_{start}}$)

Put p = 500 kPa with P_{start} = 800 kPa

$$-\frac{0.685A^*\sqrt{RT}}{V}t = ln\left(\frac{500}{800}\right) \Rightarrow t = 47.3s$$

c) Flow through nozzle is only choked while $\frac{p_o}{p} > 1.893$

Therefore when pressure in tank reaches 1.893x1.01325=1.92bar flow at nozzle will stop being choked.

$$-\frac{0.685A^*\sqrt{RT}}{V}t = ln\left(\frac{192}{800}\right) \Rightarrow t = 143.6s$$