MM2TF2 THERMOFLUIDS

Topic 6. Convective heat tranfer

Aim of this section

Heat transfer is fundamentally important for thermodynamic systems – the rate of heat transfer determines how much thermal energy is swapped from one working fluid to another. In the previous thermodynamics module, an introduction to heat transfer was presented which describes Fourier's rate of conduction heat transfer formula, and Newton's rate of convection heat transfer and the Stephan-Boltzmann rate of radiative heat transfer. Conduction heat transfer is determined by the physical property of the conducting material and its shape – this can be calculated from knowledge of these properties. Radiative heat transfer is determined by the view that two exchanging bodies have of each other and of their thermal-optical properties – this also is calculable from analysis alone. Convective heat transfer on the other hand is determined by a convective heat transfer coefficient, which depends on multiple, complex and interacting characteristics of the flow and the thermal properties of the fluid causing the heat transfer. It is complicit in many situations of thermodynamic interest, e.g. the boiler tubes in the vapour power cycle have steam and boiling water on the inside and hot gases on the outside; the evaporator and condenser tubes in a refrigeration cycle have boiling or condensing fluid on the inside and atmosphere on the outside; compressor intercoolers have hot, high pressure gas on the inside and atmospheric air or cooling liquid on the outside. Therefore it is important to firstly understand the typical magnitude of convective heat transfer coefficients on an empirical basis, and further to this, to be able to interpret the fluid mechanic and thermodynamic property conditions giving rise to the convective heat transfer from the empirical conditions, which, for common geometries and flow configurations, will yield analytical expressions representing the practically encountered heat transfer.

Aim: gain appreciation of scale of convective heat transfer in typical conditions and learn to identify and apply analytical techniques for calculating the convective heat transfer coefficient.

Objectives: revise convection heat transfer principles; recognized magnitudes of convective heat transfer coefficient in typical heat transfer fluids; establish the relationship between convection to a surface and the conduction at the surface in convective heat transfer; recognize the relationship between measuring heat flux in scaled situations and potential for application to similar situations of different scale; learn the 3 dimensionless expressions involved in natural and forced convection; develop the application of the dimensionless numbers into convection heat transfer correlation expressions for typical common flow configurations; apply the convection heat transfer correlations.

1. Revision of principle of convection heat transfer

Convection situations include both heat transfer from solid boundaries into fluids and between two mixing streams of fluids and the redistribution of internal energy within a fluid.

Convection implies the movement of fluid to transmit energy and so the study of the fluid dynamics of the situation is of particular importance.

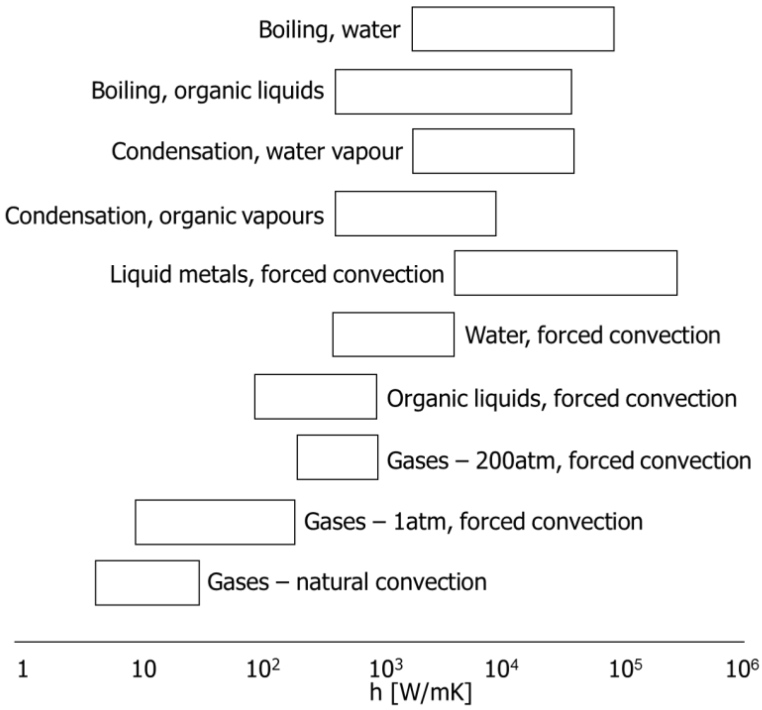
Fluid motion may be induced by buoyancy effects resulting from changes in temperatures within the fluid during the initial stages of heat transfer, which are then perpetuated as natural or free convection.

Alternatively, the fluid motion may be 'forced' by an external source - pump or compressor - and forced convection of heat will develop.

In both cases, Newton developed a formula identifying that there was a relationship between the surface area exposed to the fluid flow and the temperature difference between the surface and the bulk of the fluid which consistently showed a predictable relationship for each case:

$$\dot{Q} = hA\Delta T$$

This equations states that the *rate* of heat transfer (hence the dot above the Q) is determined by the steady temperature difference and the area by a convective heat transfer coefficient, *h*. This was great up to the point where the shape of the surface and the fluid flow conditions changed – then the value of *h* varied.



Since Newton, many researchers have investigated how to predict *h* for many different flow conditions. The results of many experiments have shown the great variation in magnitude with different heat transfer fluids as shown in the **figure from Bejan's book on Heat Transfer** (Bejan, 1993. Heat Transfer. Wiley and Sons Inc, Canada). As the type of heat transfer fluid changes, the value of *h* varies and several generic types are shown from least convective at the bottom to more convective at the top. The thing that is apparent is the vigour of the fluid can be imagined to increase in order of rising *h* as described by the various fluid conditions.

In addition to the fluid, the characteristics of the flow of fluid affect the magnitude of *h*.

Example: calculate the rate of convective heat transfer for a flat rectangular surface 1.5 m by 2.5 m, when the temperature difference between the fluid bulk temperature far from the wall is 65°C and that of the solid body behind the surface is 18°C, given that the convective heat transfer coefficient is in the range for forced convection of gases at 1 atm, and is 24W/m^{2}K.

2. Thermal resistance and convection heat transfer and overall heat transfer coefficient

It is convenient to use the analogy of electrical and thermal transfer in the form of the thermal resistance. This can be used when more than one heat transfer condition is acting on the heat transfer situation.

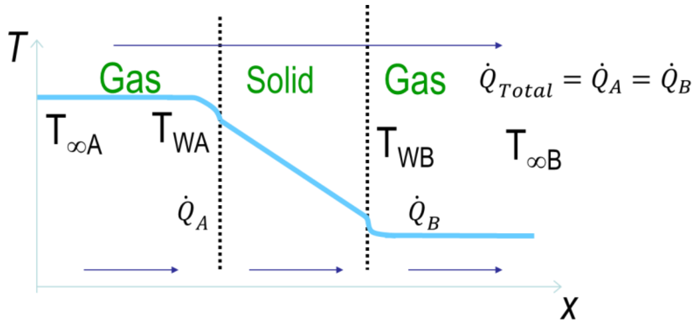
Heat transfer can be compared with Ohm's Law: I = V/R

Comparing Newton's Law of convection:

Temperature difference ΔT equivalent to potential difference ΔV

Heat flow q equivalent to Current flow I

Thermal resistance 1/hA equivalent to Electrical resistance R_{th}.



Note from the figure showing temperature between gas in a hot space on the left, that the gas interface between the solid and the gas has a non-linear temperature profile – i.e. it is not a straight line relationship between temperature and distance from the wall in the region where the temperature is changing.

Example: calculate the rate of heat transfer for a 1m length of a tube surface, when the interior temperature is 150°C and the exterior is 450°C. The external diameter is 38mm and the convective heat transfer on the external surface is in the range of forced convection for gases at 1 atm (choose a mid-range value) and the internal surface is subjected to boiling heat transfer of water (choose a mid-range value). The conductive thermal resistance is negligible because it is a steel tube with a 4mm wall thickness. For this calculation assume that the surface area is equivalent to a plane area equal to the outer circumference times the length. **NB** you will need to use the concept of thermal resistance in this calculation.

The combination of two or more heat transfer coefficients over a heat transfer surface can be solved using the method above, but there is a more convenient way to manage the final heat transfer calculation, and of expressing the combined heat transfer characteristics, by using an **overall heat transfer coefficient**. This has the symbol *U* and the same units as convective heat transfer and sums up the heat transfer on a surface due to combined effects. And so we have the overall heat transfer coefficient form of the heat transfer equation:

$$\dot{Q} = UA\Delta T_{overall}$$

The units of *U* are W/m^2K.

Example: For the previous example consider that the tube is actually in a heat exchanger and that its overall length is 45m (bent in a repeating serpentine shape for compactness). What is the overall heat transfer coefficient and what is the rate of heat transfer?

3. Analysis of how convection works

At a hot wall, the velocity is zero, and the heat transfer into the adjacent fluid takes place by conduction. Thus the local heat flux per unit area $\dot{Q}''$ is (note the notation here – the dot means rate of heat transfer and the double dash means per unit area):

$$Q'' = -k\frac{\delta T}{\delta y}\bigg|_{wall}$$

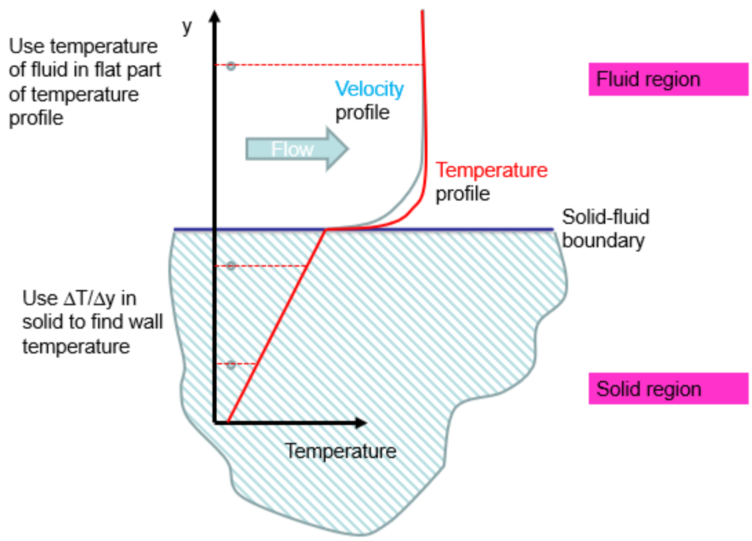


Figure showing the boundary between solid and fluid during convective heat transfer.

The conductivity is that of the fluid and the infinitesimal temperature gradient is in the very near to the wall region. The heated fluid is then carried away by convection. From Newton's law of cooling:

$$Q'' = -k\frac{\delta T}{\delta y}\bigg|_{wall} = -h(T_\infty - T_{wall})$$

note there is no area in this because once again it is double-dash Q which is per m^2 of area. Reducing the combined equation produces the following:

$$h = \frac{k\frac{\delta T}{\delta y}\Big|_{wall}}{(T_\infty - T_{wall})}$$

This indicates how the previous research managed to produce the convective heat transfer coefficients and how it can be determined from experiments for any particular configuration.

4. Nusselt number – the relation between fluid conductivity and convection.

The general situation of heat transfer is of conduction in the stationary very near wall fluid and of convective carrying away of thermal energy outside of this region. Nusselt derived a dimensionless number relating the two:

$$Nu = \frac{hL}{k_f}$$

where k is the conductivity of the fluid, and L is the representative length scale (e.g. diameter of cylinder, length of a flat plate, internal width of a duct). In this equation, we can define L and k, but *h* is unknown and so is *Nu*. Fortunately correlations have been derived for expressing *Nu* for various common flow conditions which are related to three further dimensionless numbers. The characteristic of all flow situations which affects convection rate is the ***Prandtl Number***.

Prandtl Number

All dimensionless numbers express the ratio of important defining characteristics of physical phenomena. This dimensionless group represents the ratio of the thickness of the velocity boundary layer to the thickness of the ***thermal boundary layer***. Just as there is a velocity boundary layer which defines the distance from a wall at which the velocity is 99% of the free-stream velocity far from the wall, the thermal boundary layer defines the point at which the temperature change is 99% of the temperature change from the wall to the free stream fluid temperature. The ratio is determined by the viscosity – defining the diffusion rate of momentum – and the ***thermal diffusivity*** which is the diffusion rate of thermal energy.

Thermal diffusivity, alpha

$$\alpha = \frac{k}{\rho c_p}\ [m^2/s]$$

Kinematic viscosity, nu

$$\nu = \frac{\mu}{\rho}\ [m^2/s]$$

Prandtl number, Pr

$$Pr = \frac{c_p \mu}{k}$$

Aside: an indication of the meaning of Prandtl number size.

It is a bit beyond TF2 to see exactly the relationship of Pr to boundary layer sizes, but an indicatoin is given by considering how the thermal diffusivity is involved in heating up a body by balancing heat absorbed against heat transferred:

$$mc_p \Delta T_{time} = \frac{kA}{\Delta x} \Delta T_{space}$$

$$m = \rho x A$$

$$\frac{\rho x A c_p}{k} \Delta T_{time} = \frac{A}{\Delta x} \Delta T_{space}$$

$$\frac{1}{\alpha} \propto \frac{\Delta T_{space}}{\Delta x}$$

i.e. the gradient of temperature in space decreases as α increases

Similarly, kinematic viscosity involved in friction in the fluid causes velocity gradient for a given wall shear stress in the fluid:

$$\tau_{wall} = \mu \frac{\Delta u}{\Delta x}$$

$$\frac{1}{\nu} \propto \frac{\Delta u}{\Delta x}$$

i.e. the gradient of velocity in space decreases as kinematic viscosity increases causing a velocity gradient

Therefore high Pr means that the ratio of gradient of temperature to gradient of velocity is large, and if the gradient is large it means that the boundary layer is thin (because temperature changes more quickly with distance with high gradient). This implies that the fluid is not very thermally conductive and quite viscous. Generally if Pr < 1, the thermal boundary layer is thicker than the velocity boundary layer and when Pr > 1 the opposite is the case.

Example: Using the values from the tables on p.10 for further properties of water and stem, for dynamic viscosity, density, thermal conductivity and specific heat capacity, calculate the thermal diffusivity of water and kinematic viscosity, and hence the Prandtl number for water at 20°C, and compare with the value stated in the tables. Compare this with the Prandtl number for air on p.16. What can be said about the thermal properties of the two fluids from this?

5. Influence of turbulent flow in forced convection

Reynolds Number

Forced convection defines situations in which the fluid is driven against the wall by mechanical means – a forcing of the fluid to flow. In cases where the velocity is thus defined by a flow situation, the Reynolds Number reflects the amount of mixing of the fluid takes place, which in turn affects the convection. Where there is less turbulence and the Re is low, the mixing is low and convection will be correspondingly low. Conversely where Re is high, convection is high. In cases of forced convection therefore, correlations can be found relating Re and Pr to Nu and this will define the heat transfer coefficient.

Example: A radiator pipe has an internal diameter of 5mm, and the flow rate of hot water is 30 g/s. The bulk temperature of the water is 65°C. Using the following correlation, derive the heat transfer coefficient at the internal pipe wall. Comment on where this fits into the chart of *h* values earlier presented.

$$Nu = 0.023Re^{0.8}Pr^{0.4}$$

6. Grashof Number

Natural convection defines situations where there is no mechanical motivation of the fluid – but natural stratification of the temperature field caused by heating or cooling of surfaces creates a natural circulation of fluid. This is known in common parlance as 'hot air rises' – but is not limited to air only. The driving force is the alteration of density due to high density in cold regions and low density in warm regions. It is naturally difficult to derive the velocity of the fluid because it is entirely free in the body of the fluid. Although derivations can be sought for the velocity field, it is not really required to work out the heat transfer. It is made even more difficult to determine the velocity field because of the nature of the velocity profile according to the simulation from a BEng project below which shows that although the temperature reduces monotonically, the velocity is zero at the wall (on the left) and grows before falling back towards zero in the far from wall position. And this is at one height only – so if velocity were used to determine the convection, it would have to be an average over the height over which convection was done.

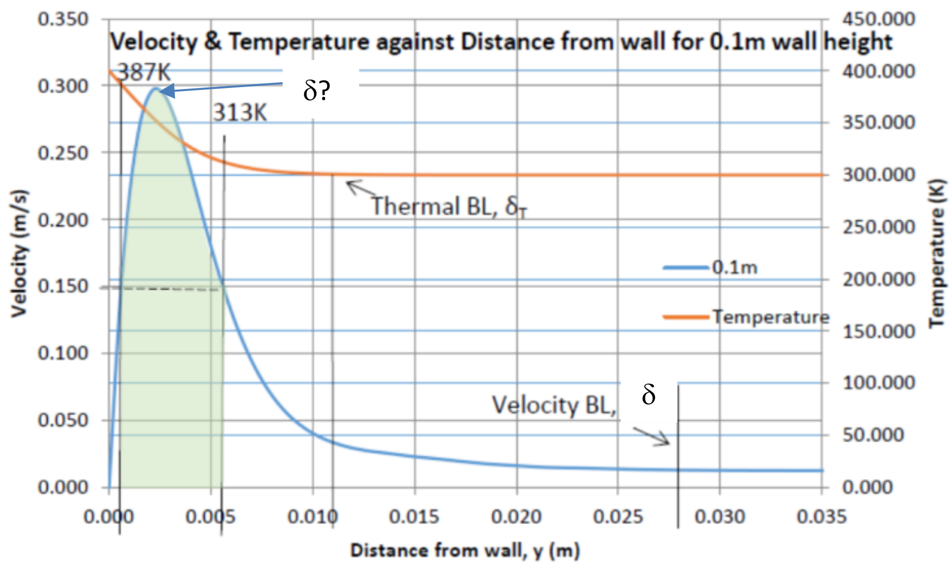
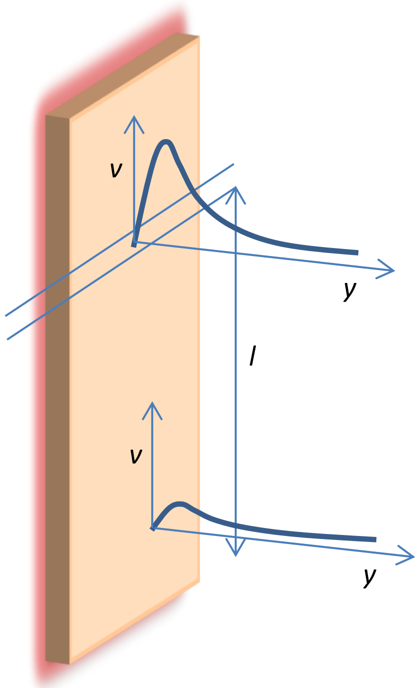


Figure showing velocity profile and temperature profile for a hot wall at 400K on y=0 at a height of 0.1m from the beginning of the heated section. (M.Brown, BEng thesis 2014)



Grashof defined a dimensionless group which represents the ratio of the buoyancy force to the viscous resistance force, and defines the level of turbulence caused by a natural convection flow. It is only dependent on the temperature differences present.

$$Gr = \frac{g\beta l^3 \Delta T}{\nu^2}$$

In this formula, it is necessary to understand all the terms. The only obvious ones are *g* the local gravitational constant and ΔT, the difference in temperature between the bulk fluid temperature and the temperature of the surface. β is the ***compressibility*** of the fluid, and for gases it is defined as $1/T_f$, and T_f is referred to as the ***film temperature*** and is the average of the wall temperature and the bulk fluid temperature far from the wall. The length *l* is the distance from the start of the hot surface, and defines the Grashof number at a particular height. ν is the kinematic viscosity, but it is at the film temperature for this calculation, since it varies strongly with temperature.

Because of the square term on the viscosity, which is generally a very small number, Gr is usually very large for common liquids and gases. A particular situation will have a Gr which is either large representing turbulent buoyant flow or small representing laminar buoyant flow. In cases of natural convection, the strength of the convection is determined by the value of the Grashof number and a correlation relating Gr and Pr to Nu can be found for common configurations in texts.

The hot plate in this case shows that as height increases the velocity intensity increases, which has a corresponding increase in mixing intensity (turbulence). The flow will be laminar up to a point and beyond that turbulent. The transition is determined by the Grashof number, and a condition will be stated for particular geometrical arrangements.

A mean Nusselt number correlation can usually be found from the texts, but it is important to make sure that it is not the Nu for a particular height and is the mean. If it is just the Nu for a particular height then it must be integrated over the entire height and an average value determined. Only mean values will be used in this course.

Example: Calculate the Nusselt number and hence the convective heat transfer coefficient for a flat wall having height 2m, and a temperature of 400°C in an ambient air environment at 20°C. Use the following correlation:

$$Nu = 0.59(GrPr)^{0.25}$$

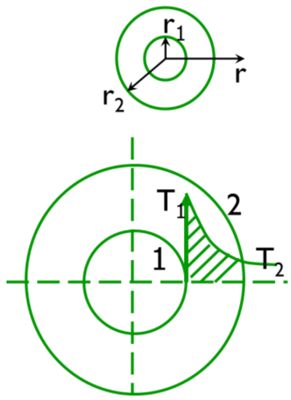
For 10^3 < GrPr <10^9, and:

$$Nu = 0.13(GrPr)^{0.33}$$

For 10^9 < GrPr <10^{12}.

7. Application of the axisymmetric wall conduction equation

From the previous module on thermofluids, the idea of conduction radially through an axisymmetric pipe was introduced. In this case it must be remembered that instead of a constant cross section through which heat is conducted, as the heat moves radially outwards, the area through which it is conducting is increasing, as indicated in the figure.



For this case it was shown that the conduction through the pipe wall can be expressed by integrating the heat transfer across each finite radial annulus:

$$\dot{Q}' = -kA\frac{dT}{dr} = -k2\pi r\frac{dT}{dr}$$

Where the single dash implies 'per unit length of pipe', i.e. per m length. Given constant k through the material, then:

$$\frac{\dot{Q}'}{2\pi k}\frac{dr}{r} = -dT$$

and integrating between inner wall, 1 and outer wall, 2, gives:

$$\frac{\dot{Q}'}{2\pi k}ln\frac{r_2}{r_1} = -(T_2 - T_1)$$

And the thermal resistance of the pipe wall per metre is therefore:

$$R = \frac{ln\frac{r_2}{r_1}}{2\pi k}$$

Example: calculate the conductive thermal resistance of a steel pipe, conductivity 20W/mK, of internal radius 30mm and wall thickness 5mm which is insulated by a 150mm thick jacket of material with a conductivity of 0.5W/mK.