MM2TF2 THERMOFLUIDS

Topic 4. Reciprocating air compressors – how to make energetic air

Aim of this section

Air at medium pressures (up to 40 bar) is useful as an energy source for pneumatic powered tools as used in manufacturing, and for pneumatic suspensions, and this is often supplied from reciprocating compreesors. Reciprocating vapour compressors, akin to air compressors, are used commonly in refrigeration circuits.

Aim: learn to calculate work and heat exchanges and the air delivery expected from reciprocating gas compressors

Objectives: understand reciprocating compressor terminology; analyse *machine* cycle as opposed to *closed* cycle process; calculate the volumetric displacements and efficiency due to polytropic compression and expansion; calculate the work and heat transfers and understand how the heat transfer in the cylinder determines the polytropic index; understand and calculate the various efficiency measures of reciprocating compressors; understand the reason for intercooling.

Thermo topic 7: Reciprocating air compressors

1. Reciprocating Compressors.

These are reciprocating piston machines typically needed to provide a supply of compressed air or vapour. A motor is used to drive the compressor with mechanical work, which is transferred into compression work on the gas. Gas is sucked into the ingoing duct and expelled at a higher pressure, usually to a pressure vessel. When the pressure vessel reaches the required pressure, the compressor will stop.

Described as:

- Single or double acting (compression on one or both sides of piston)
- Single or multistage (number of stages of compression before air is delivered)
- Positive displacement (cylinder inlet and outlet valves needed to prevent backflow)
- Self acting valves (opened and closed by pressure differentials across the valves).

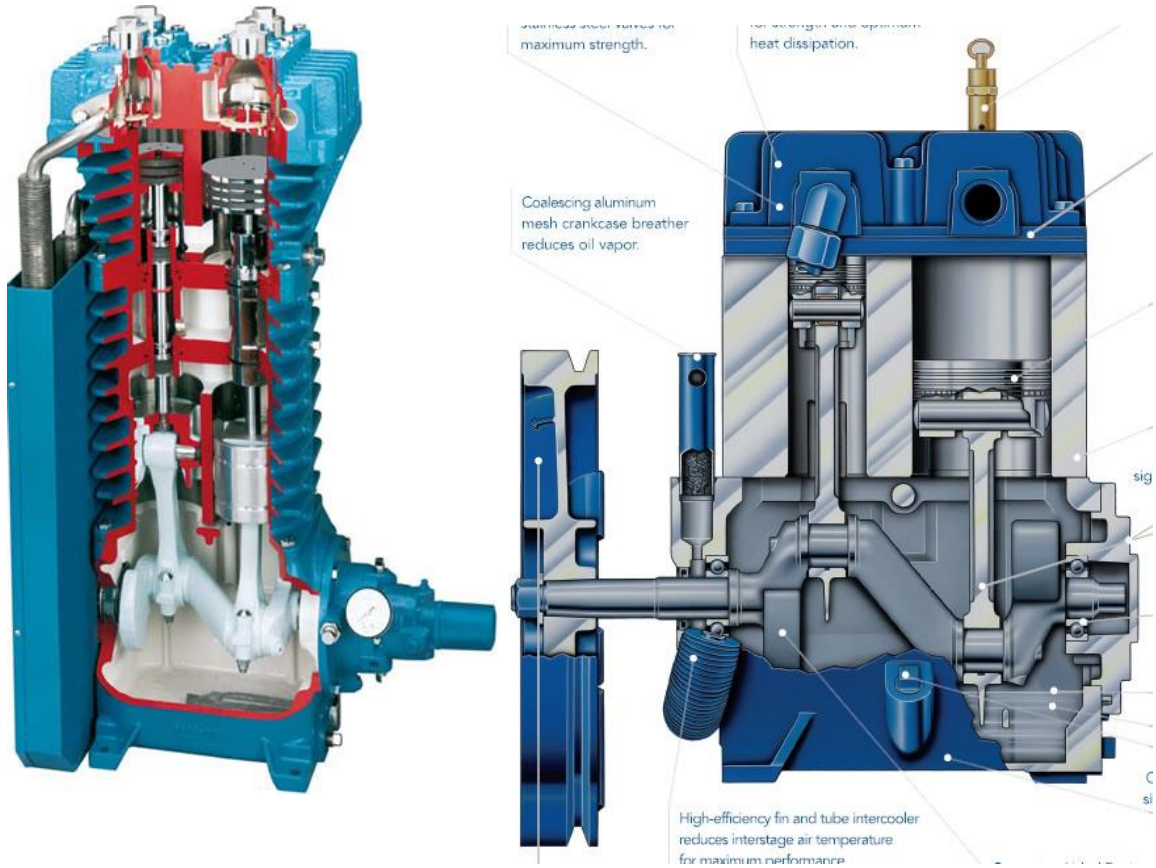
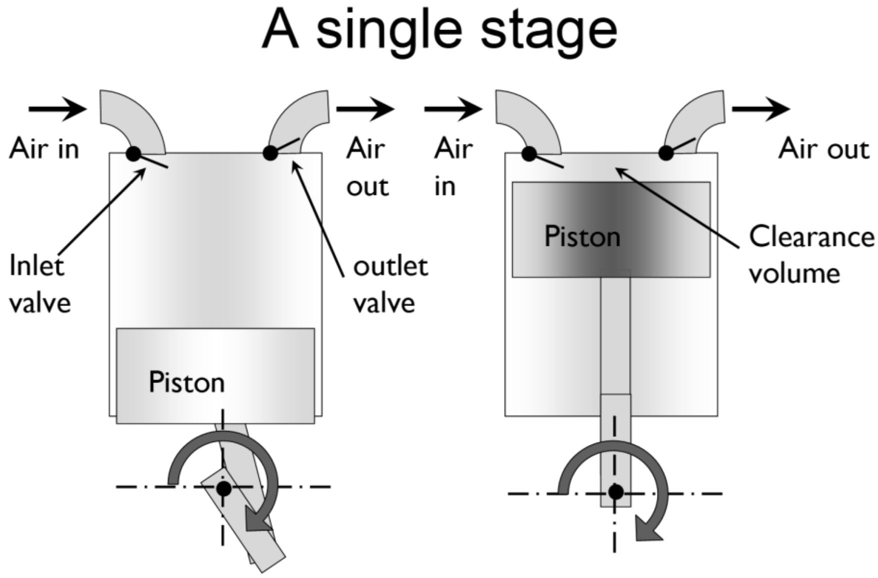


***Figure** showing on the left a two stage, double acting compressor – larger cylinder on right which compresses both on the upstroke and the down stroke followed by a smaller cylinder on the left which receives the gas from the low pressure piston, which has smaller specific volume and therefore smaller volume cylinder for same mass rate. On the right, a two stage single acting compressor – similarly low pressure on the right cylinder and high pressure on the left.*

2. Single stage compression

The figure shows a single stage compressor, with a piston driven up and down the cylinder by a con-rod and drive shaft. Air is drawn in when the piston moves down and the valve on the left opens inwards. This valve cannot open outwards. When the piston drives up then the valve on the right will open outwards – it cannot open inwards. So the air can only travel in one direction through the compressor. Several terms need to be remembered as follows:

Stroke – distance from Bottom Dead Centre (**BDC**) to Top Dead Centre (**TDC**)



Swept volume – volume swept through from BDC to TDC.

Clearance volume – a small volume which the piston does not drive through at the top of the cylinder, approximately 10% of swept volume typically. It allows the valves to be clear of the piston smashing into them.

Machine cycle – the gas goes through a machine cycle rather than a full thermodynamic cycle because it is drawn in and exhausted part way through the cycle, rather than just compressed and expanded so there is a useful delivery out of the machine – a high pressure gas.

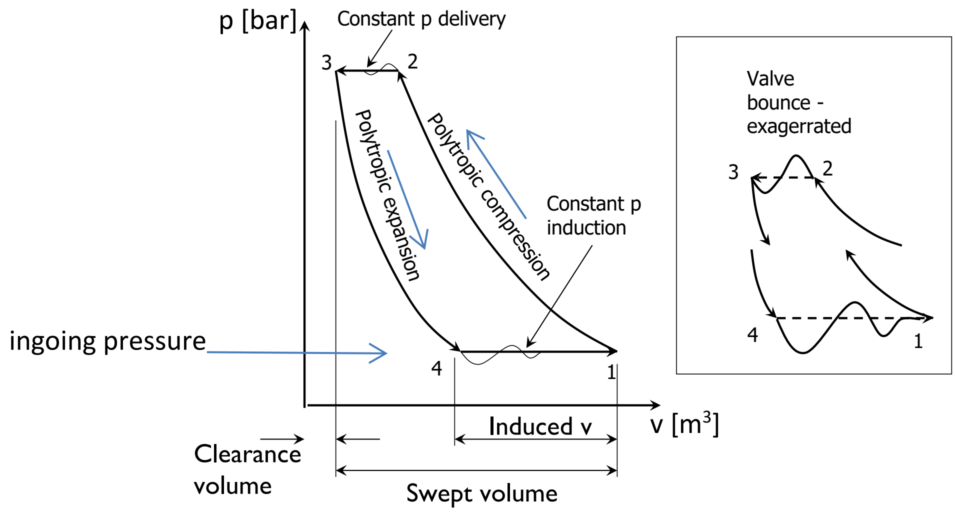
Automatic valves – spring return.

3. Machine cycle and polytropic processes

The figure on the next page shows a pressure vs. volume diagram. It looks usual regarding the compression and expansion, but what is different to a closed cycle is that at the bottom of the graph, the inlet valve opens at point 4 – this is when the pressure in the cylinder has reached the ingoing pressure – for example atmospheric pressure if this is just a single stage air compressor. Notice that the piston has moved from the clearance volume at point 3, where a small volume of gas was at high pressure; it is not until the piston has travelled quite far down the cylinder at point 4 (looking at the v axis) when the pressure has fallen to a point where the non-return valve on the inlet can open.

The second unusual thing is that at the low pressure, the pressure remains constant while the piston moves further down from point 4 until point 1 at BDC. This means that new air is being sucked into the cylinder by the piston movement. Then compression occurs up to point 2. And the piston still has a lot of travel left at this point, but instead of the pressure increasing, the outlet valve opens letting air out of the cylinder. As the piston moves up to TDC position, air is expelled from the piston to the high pressure chamber waiting to receive the gas. Then at TDC, the outgoing valve closes and the cycle starts again.

Note that this is not a closed cycle – it is an open cycle! The compression and expansion processes are closed processes, but the overall result is a through flow of the gas.

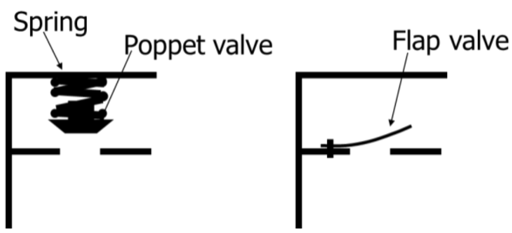


To the right of the figure there is a inset highlighting what happens when the valves open and close. Because the pressure at which the valve opens is arrived at reasonably suddenly, and the pressure on both sides of the valve becomes equal at that point, the valve flutters on its spring mounting. This is called **valve bounce**.

If the polytropic processes occurred rapidly and well insulated, then they are near adiabatic because there is not much scope for heat transfer from the gas. If the process is done slowly and full heat transfer is permitted, then the process will be near isothermal. Hence in the middle must be polytropic: pv^n = constant, and $1<=n<=\gamma$. Typical value of n is 1.3.

The ideal (indicated in the diagram) and actual indicator diagrams differ because of imperfect valve operation, leakage past piston, non-constant polytropic index, *n*, during compression and expansion.

Valve bounce can often cause pressure oscillations on the 'constant pressure' processes, as the valve bounces on its springy mounting. The valves may be constructed such as indicated in the figure below.



4. Work done if only gas compression is considered – no friction etc.

Reversible (ideal) work done (by compressor on gas is positive) for the equivalent steady flow open system is according to the formula:

$$\dot{W} = \dot{m}(h_2 - h_1) - \dot{Q} = \dot{m}\int_1^2 dh - \dot{Q}$$

for reversible process:

$$\dot{Q} = \dot{m}\int_1^2 Tds$$

and

$$\int_1^2 Tds = \int_1^2 (dh - vdp)$$

therefore:

$$\dot{W} = \dot{m}\int_1^2 dh - \dot{m}\int_1^2 (dh - vdp) = \dot{m}\int_1^2 vdp$$

and we know that the pressure-velocity relationship of the gas is according to:

$$pv^n = c$$

so we can derive the work for the compression process from these equations:

$$dw = \int_1^2 v.dp = \int_1^2 \frac{c^{1/n}}{p^{1/n}}.dp = \frac{1}{1 - 1/n} c^{1/n}\left[p^{1-1/n}\right]_1^2 = \frac{n}{n-1} c^{1/n}\left[p_2^{1-1/n} - p_1^{1-1/n}\right]$$

actually $p_1v_1^n = p_2v_2^n = c$, therefore:

$$c^{1/n} = p^{1/n}v$$

and:

$$dw = \frac{n}{n-1}\left[p_2^{\ 1/n}v_2p_2^{\left(\frac{n-1}{n}\right)} - p_1^{\ 1/n}v_1p_1^{\left(\frac{n-1}{n}\right)}\right] = \frac{n}{n-1}[p_2v_2 - p_1v_1]$$

where v (m^3/kg) is the specific volume and w is the specific work (J/kg).

So for the work rate:

$$\dot{W} = \dot{m}p_1v_1\frac{n}{n-1}\left[\frac{p_2v_2}{p_1v_1} - 1\right]$$

or in terms of pressure ratio only using $pv^n=c$ again:

$$\dot{W} = \dot{m}p_1v_1\frac{n}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

alternatively, using the gas law pv=RT

$$\dot{W} = \dot{m}R\frac{n}{n-1}[T_2 - T_1]$$

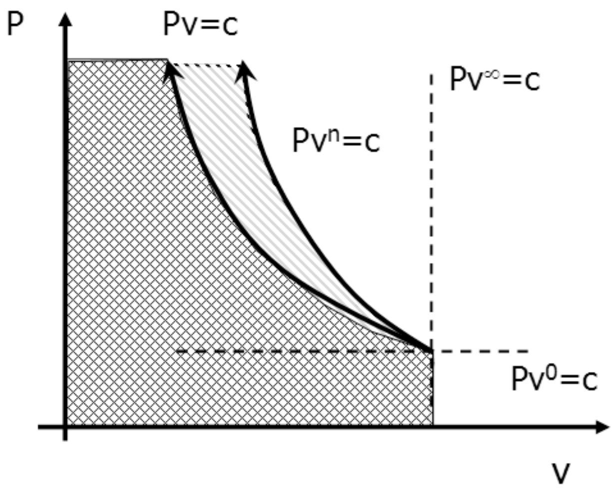
it just depends where you want to do your calculation of polytropic conditions – you either need T_2 and T_1 or v_2 and v_1. In order to calculate temperature, the T-p equation for polytropic compression is required – which will be on a formula sheet as:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Example: what is the temperature of a gas which undergoes compression from 1 bar, 20°C up to 6 bar when the polytropic index is 1.3?

What is the work done in a compressor which delivers this air at 6 bar at a rate of 1.5 g/s?

in Europe work is negative for work out of the fluid, i.e. USA is useful output, Europe is strictly about what the fluid experiences



5. Ideal work (i.e. no friction) and optimum work

Less work is the better for a compressor – it is the effort required to deliver the air to the higher pressure. The equation above showed that the magnitude of the work decreases as *n* decreases – the limiting point is *n*=1, i.e. isothermal process. That would be the case where all thermal energy generated in the gas was removed as it was generated in the gas. This, *isothermal work*, is the minimum work to compress the air to the higher pressure and therefore the optimum situation. We have the actual work done – with the polytropic index – somewhere between adiabatic and isothermal. The cases are illustrated in the p-v diagram above, which shows that as the polytropic increases from the minimum at isothermal process to the maximum when there is no heat transfer (i.e. perfectly insulated compressor $n=\gamma$) the work which is the area under the line increases.

In order to approach isothermal, economically sensible efforts are made to cool the compressed gas in the cylinder, and at any other points where it can be cooled.

6. Heat transfer to the cooling jacket which is put on the cylinder

The heat transfer to the jacket is what decides the polytropic index – if perfectly thermally insulated, then gamma, if constant temperature conditions are caused by the jacket then 1, otherwise a value somewhere between the two, n. Polytropic and isothermal processes require heat transfer to the jacket around the compression chamber. For a polytropic process we can calculate the heat transfer using the steady flow energy equation. Therefore, the work input, and the enthalpy rise associated with it, require that this heat is lost to the cooling jacket surrounding the cylinder. This can be calculated by the formula as follows.

The first law requires that if work is done, there will be heat loss. And this is expressed from the SFEE:

$$\dot{Q} + \dot{W} = \dot{m}\Delta h$$

using the previous expression for work and knowing $h=mc_p\Delta T$:

$$\dot{Q} = \dot{m}c_p(T_2 - T_1) - \dot{m}R\frac{n}{n-1}[T_2 - T_1]$$

Now we know that from $c_p=c_v+R$, that $R=c_v(\gamma-1)$:

$$\dot{Q} = \left\{\dot{m}c_v\gamma - \dot{m}c_v(\gamma-1)\frac{n}{n-1}\right\}[T_2 - T_1]$$

$$\dot{Q} = \dot{m}\frac{\gamma - n}{1-n}c_v[T_2 - T_1]$$

Example: Calculate the heat transferred to the jacket from the previous example when air is used.

7. Isothermal efficiency

Since we have a compression which has an *ideal* work (whereby ideal means no friction taken into account) which has heat transfer from the cylinder, which in turn gives rise to the polytropic index which in turn means more work from the motor driving the compressor than is the case for the optimal isothermal conditoin – therefore we'd like to know how far from 'the optimal' it is. The best would be isothermal. So we can compare the actual *ideal-no-friction* work with the *ideal-isothermal-no-friction* case. We know the actual work, so what about the isothermal work? It is:

$$dw = \int_1^2 v.dp = \int_1^2 \frac{c}{p}.dp = c[\ln p]_1^2 = c\left[\ln\frac{p_2}{p_1}\right]$$

Therefore for the work rate:

$$\dot{W} = \dot{m}RT_1\left[\ln\frac{p_2}{p_1}\right]$$

Isothermal efficiency is therefore:

$$\eta_{isothermal} = \frac{\dot{W}_{isothermal}}{\dot{W}_{actual-ideal}} = \frac{\dot{m}RT_1\left[\ln\frac{p_2}{p_1}\right]}{\dot{m}R\frac{n}{n-1}[T_2 - T_1]}$$

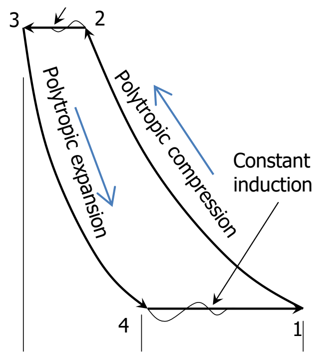
Example: for the previous example, calculate the isothermal efficiency.

8. Volumetric efficiency

The clearance volume is a small volume of gas at the top of the cylinder which is compressed and then expanded to the lower pressure before new gas can be drawn into the cylinder. Because it has to expand before new gas can be drawn in, there is a loss of use of the stroke of the piston, and this can be usefully quantified as follows:

$$\eta_{vol} = \frac{volume\ induced\ at\ initial\ state}{swept\ volume\ of\ piston}$$

Refer to the figure for numbers around the cycle.



$$\eta_{vol} = \frac{v_1 - v_4}{v_1 - v_3}$$

There is the difficulty of calculating v_4 to be overcome, but the other volumes are known.

v_4 is found by applying polytropic expansion calculation from formula sheet as follows:

$$\frac{v_4}{v_3} = \left(\frac{p_2}{p_1}\right)^{1/n}$$

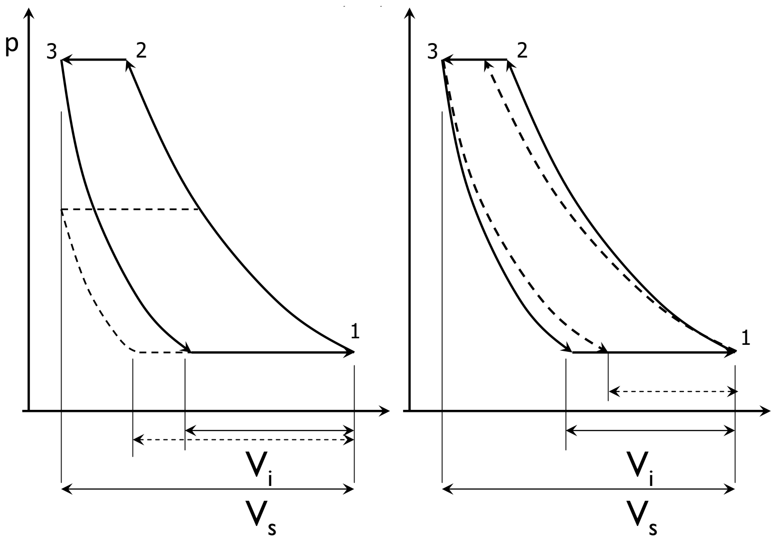
therefore:

$$\eta_{vol} = \frac{v_1 - v_3\left(\frac{p_2}{p_1}\right)^{1/n}}{v_1 - v_3}$$

Or writing v_1-v_3=v_s and v_3=v_c for the swept and clearance volumes respectively:

$$\eta_{vol} = \frac{v_s + v_c - v_c\left(\frac{p_2}{p_1}\right)^{1/n}}{v_s} = 1 - \frac{V_C}{V_S}\left[\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right]$$

This volumetric efficiency is affected by the polytropic index, *n* and by the pressure ratio, p_2/p_1. The figure helps to show why this is so; the dotted lines show the effects. In the first case, on the left, where the pressure ratio is increased, the swept volume is constant and the expansion has to take up more of it in the case of the solid lines, whereas the dotted lower pressure case shows that air starts to be induced earlier. In the second case on the right, reducing *n* causes the expansion and compression lines to lean more towards the horizontal (i.e. towards pv^0 – constant pressure process) and this also causes the induced volume to decrease.



Example: a single stage air compressor has a pressure ratio of 6 and a piston diameter and stroke of 5 cm. The polytropic index is 1.3. The clearance volume is 10% of the swept volume. What is the volumetric efficiency?

9. Multistage compression

The effect of using a single stage to try to attain a high pressure has just been shown to result in increasingly poor volumetric efficiency. As delivery pressure is raised, volumetric efficiency falls and mass flow rate falls (because the induced volume decreases – as seen earlier). The formula for volumetric efficiency shows that it is zero when p_2/p_1=22.6 for n=1.3 and V_c/V_s=10%. In practice, (p_2/p_1) for a single stage is limited to approximately 4:1, and two or more stages are used for higher delivery pressures. The pressure between stages is the intermediate pressure p_i (which is now the inlet pressure for the second stage).

Each stage can be treated as a separate subject.

There must be conservation of mass.

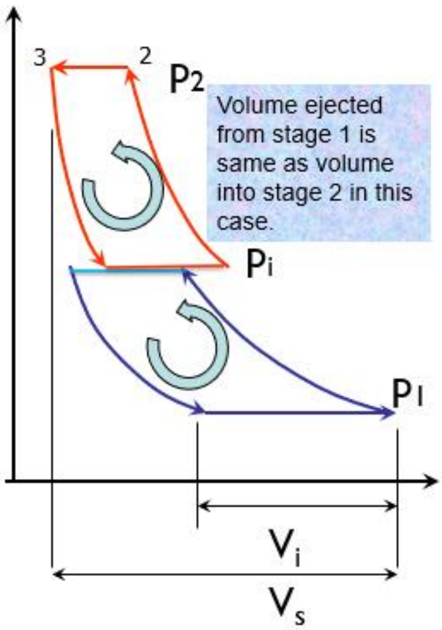
The pressure from the outlet of stage 1 is the inlet pressure for stage 2.

It can be shown that the total work on the compressor to drive the process is a minimum when the intermediate pressure is:

$$p_i = \sqrt{p_1 p_2}$$

And in this case W_{LP}=W_{HP}.

The practical way of making a multi stage compressor requires that there is a volume of air between each pair of

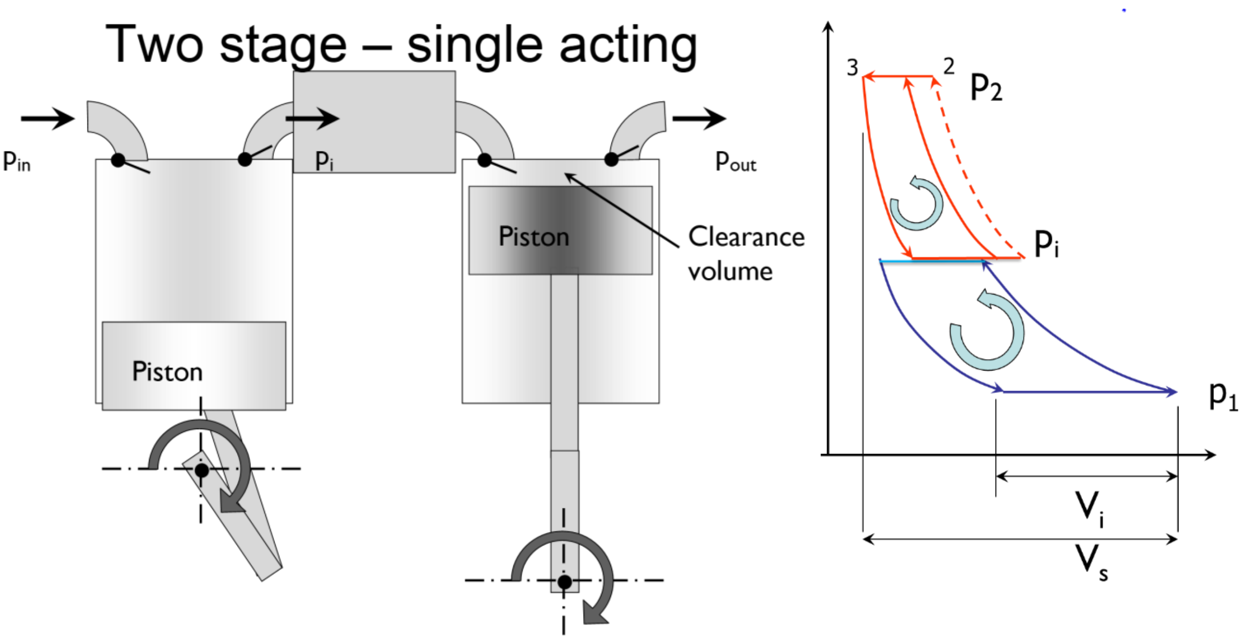


stages, the pressure of which does not alter much as the valves open and close at slightly different times when there are more than two stages, and that even in the case of two only, it would be no good to try to fill the high pressure cylinder with the delivery from the first stage when there is a smooth delivery expected – the pressure would oscillate wildly. The inter-stage volume is illustrated in the figure. It provides an opportunity – here is a perfect place to cool the air down between the stages, to attempt to get closer to isothermal compression.

10. Inter-stage cooling

Intercooling is usually done by passing cold water through a cooling coil in the intermediate receiver chamber. Usually the air is cooled between stages to minimise the specific work input (isothermal compression would be best).

The intercooler reduces the air temperature from L.P. delivery temperature to, ideally, T_1, the compressor inlet temperature. So for an ideal, two stage compressor with complete intercooling (T_i=T_1). The effect is seen in the figure below.



Reducing the temperature reduces the specific volume at intermediate pressure, ideally to T_1. The area under the dashed line is the work saved by intercooling.

It is possible to make a very compact compressor using the downstroke to drive the second stage as shown in the figure on the following page. This shows that the downstroke is used on a smaller diameter cylinder to increase pressure to second stage higher pressure, and that an inter-stage cooling volume is adjacent to the compressor cylinder to with a water tube heat exchanger to make the work required to drive the compressor less.

11. Mass flow of air delivery

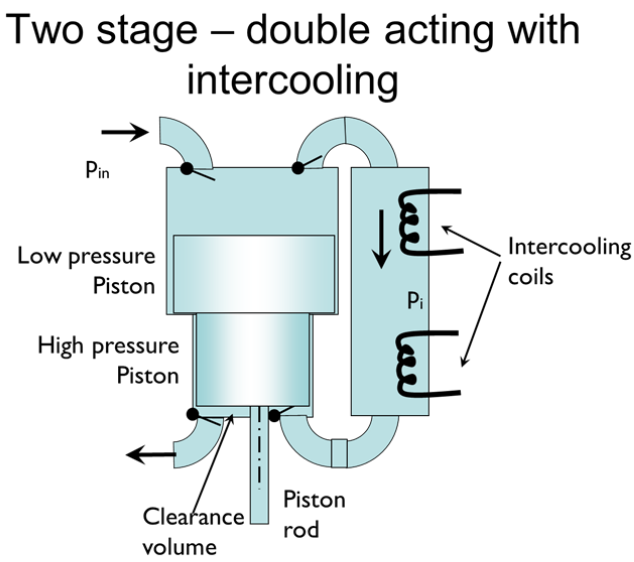
Of course the main reason for the compressor is the delivery of air at high pressure and we want to be able to calculate that. The calculation is based on the first stage, since we know the incoming condition and the details of the cylinder and piston operation of that stage.

Per cycle: $m_{induced}=\eta_v.V_s.\rho_1$

Where η_v is the volumetric efficiency, V_s is the swept volume, and ρ_1 is the density of the ingoing gas.

Per second: $m=N.\eta_v.V_s.\rho_1$

N is the number of cycles of the compressor per second.



Where N is the number of strokes of the first stage piston, which for a compressor with a single inlet stage is the motor revolutions per minute (r.p.m.) divided by 60 for strokes per second. If the compressor has a double acting compressor (compresses on up and down stroke of the piston), then for the case where it is single stage, it means that both the upstroke and the downstroke are intake strokes, and the intake of each of the two compressions should be considered, so there will be twice the amount of mass flow (assuming the volume taken up by the compressor's connecting rod in the driving side is negligible).

Example: calculate the mass flow rate for atmospheric air at 1 atm drawn into a single acting compressor with volumetric efficiency of 80%, and a swept volume of 200ml and a cycle rate of 300 cycles per minute.