$$\frac{dM}{dt} = \sum \dot{M}_{in} - \sum \dot{M}_{out} = \dot{M}_{in,1} + \dot{M}_{in,2} - \dot{M}_{out,1} - \dot{M}_{out,2}$$

$$M(x,y,t) = \rho(x,y,t)\, dx\, dy \cdot 1$$

$$\frac{dM}{dt} \rightarrow \frac{\partial \rho}{\partial t}\, dx\, dy$$

$$\dot{M}_{in,1} = \rho u\, dy$$

$$\dot{M}_{in,2} = \rho v\, dx$$

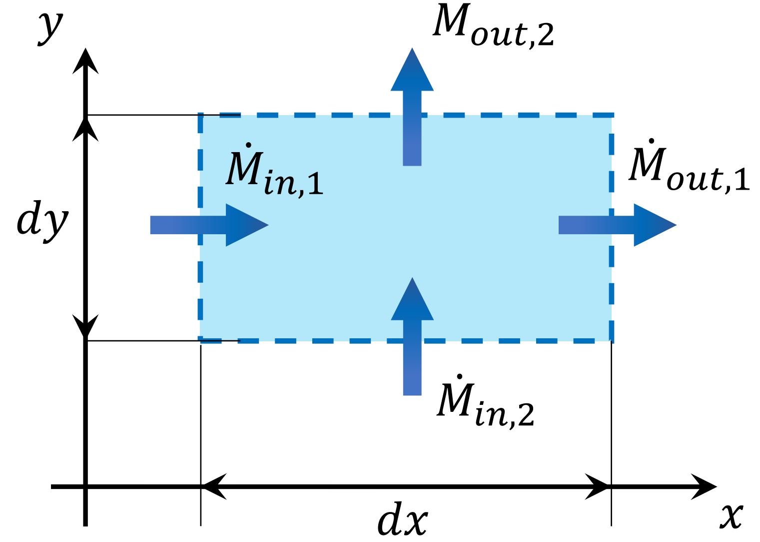
$$\dot{M}_{out,1} = \dot{M}_{in,1} + d\dot{M}_{in,1} = \dot{M}_{in,1} + \frac{\partial \dot{M}_{in,1}}{\partial x} dx + \cancel{\frac{\partial \dot{M}_{in,1}}{\partial y} dy} + \cancel{\frac{\partial \dot{M}_{in,1}}{\partial t} dt}$$

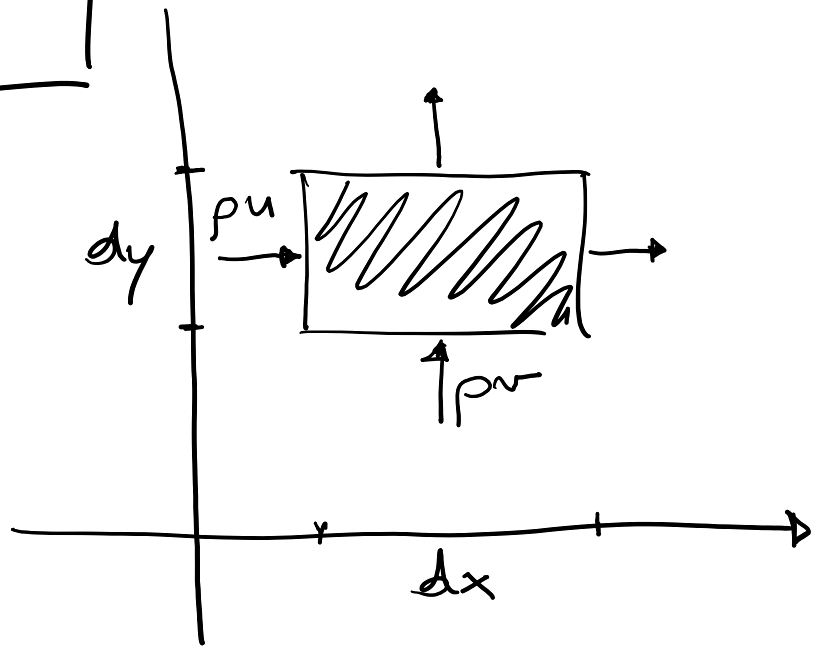
$$= \rho u\, dy + \frac{\partial}{\partial x}(\rho u\, dy)\, dx = \left[\rho u + \frac{\partial (\rho u)}{\partial x} dx\right] dy$$

$$\dot{M}_{out,2} = \left[\rho v + \frac{\partial (\rho v)}{\partial y} dy\right] dx$$

$$\left[\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y}\right] \cancel{dx\, dy} = 0$$

$$\boxed{\cancel{\frac{\partial \rho}{\partial t}} + \rho \frac{\partial \cancel{\rho} u}{\partial x} + \rho \frac{\partial \cancel{\rho} v}{\partial y} = 0}$$

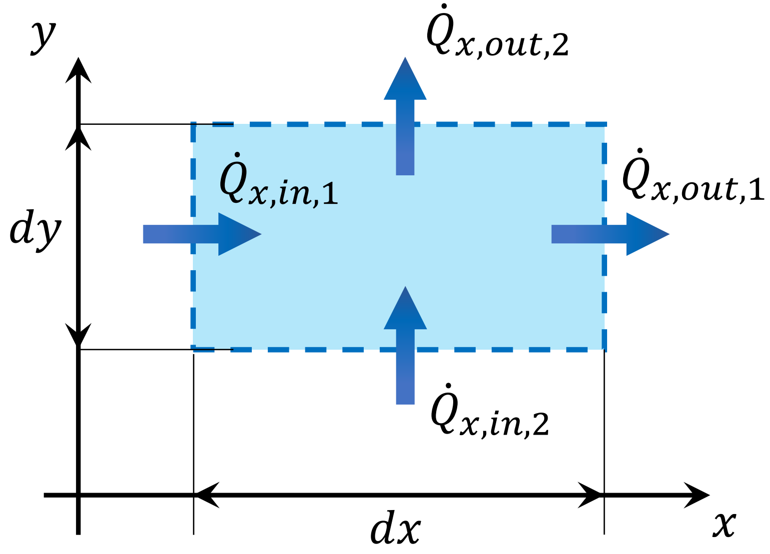




$$\sum F_x = \sum \dot{Q}_{x,out} - \sum \dot{Q}_{x,in} + \frac{dQ_x}{dt} = *$$

$$Q_x(x,y,t) = \rho u\,dx\,dy$$

$$\frac{dQ_x}{dt} \rightarrow \frac{\partial Q_x}{\partial t} = \frac{\partial(\rho u)}{\partial t}\,dx\,dy$$

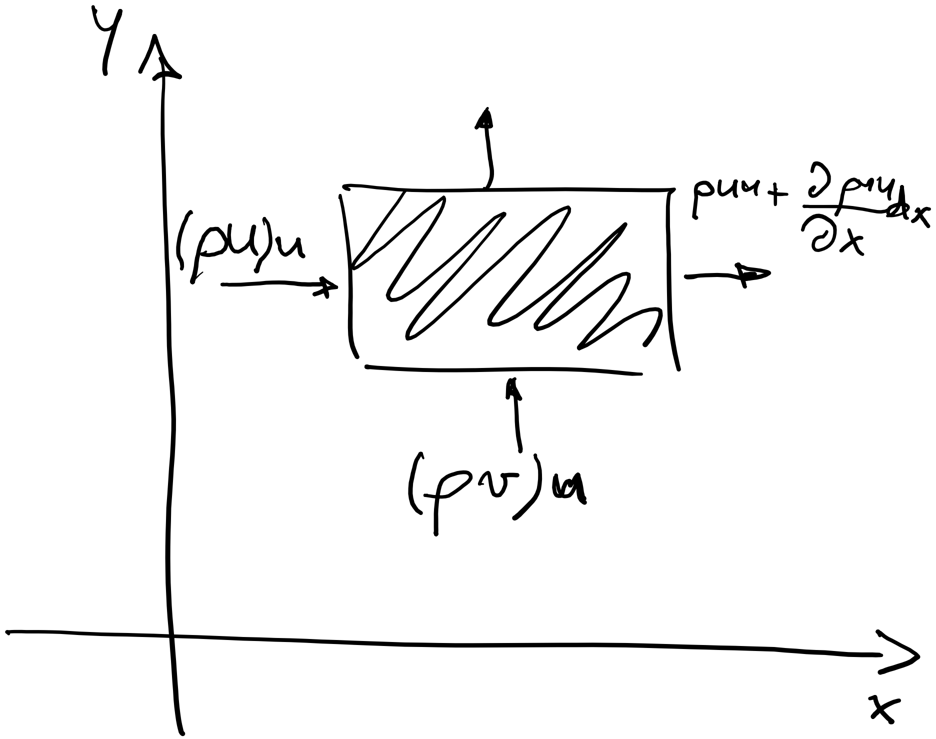


$$\dot{Q}_{x,in,1} = \rho u u\,dy$$

$$\dot{Q}_{x,in,2} = \rho v u\,dy$$

$$\dot{Q}_{x,out,1} = \left[\rho u u + \frac{\partial \rho u u}{\partial x}\,dx\right] dy$$

$$\dot{Q}_{x,out,2} = \left[\rho v u + \frac{\partial \rho v u}{\partial y}\,dy\right] dx$$



$$* \sum F_x = \left[\frac{\partial \rho u}{\partial t} + \frac{\partial(\rho u u)}{\partial x} + \frac{\partial \rho v u}{\partial y}\right] dx\,dy$$

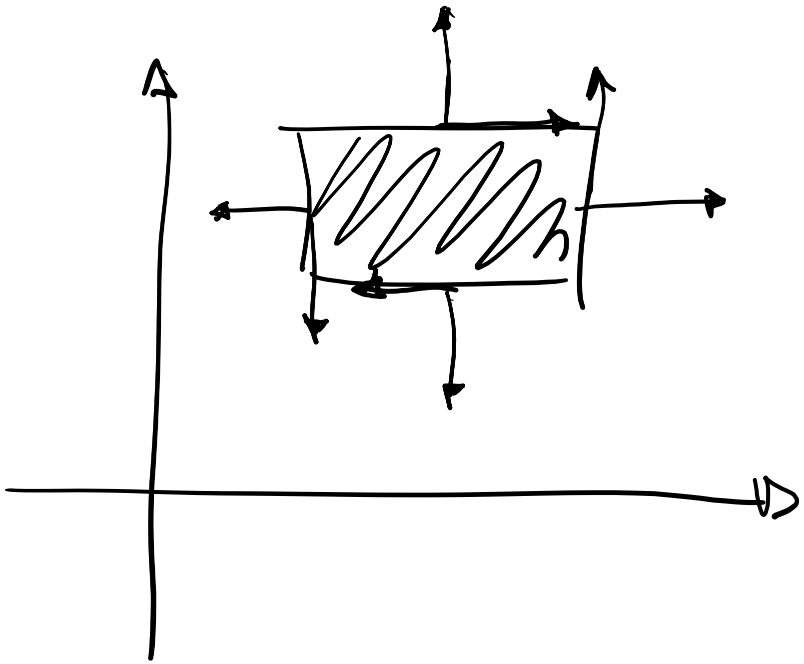
$$\boxed{\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} + \frac{\partial \rho v u}{\partial y}} = \rho \frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial t} + \rho u \frac{\partial u}{\partial x} + u \frac{\partial \rho u}{\partial x} + \rho v \frac{\partial u}{\partial y} + u \frac{\partial \rho v}{\partial y} =$$

$$= \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \cancel{u \underbrace{\left(\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} \right)}_{=0}}$$

$$\boxed{\sum F_x = \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) dx\, dy}$$

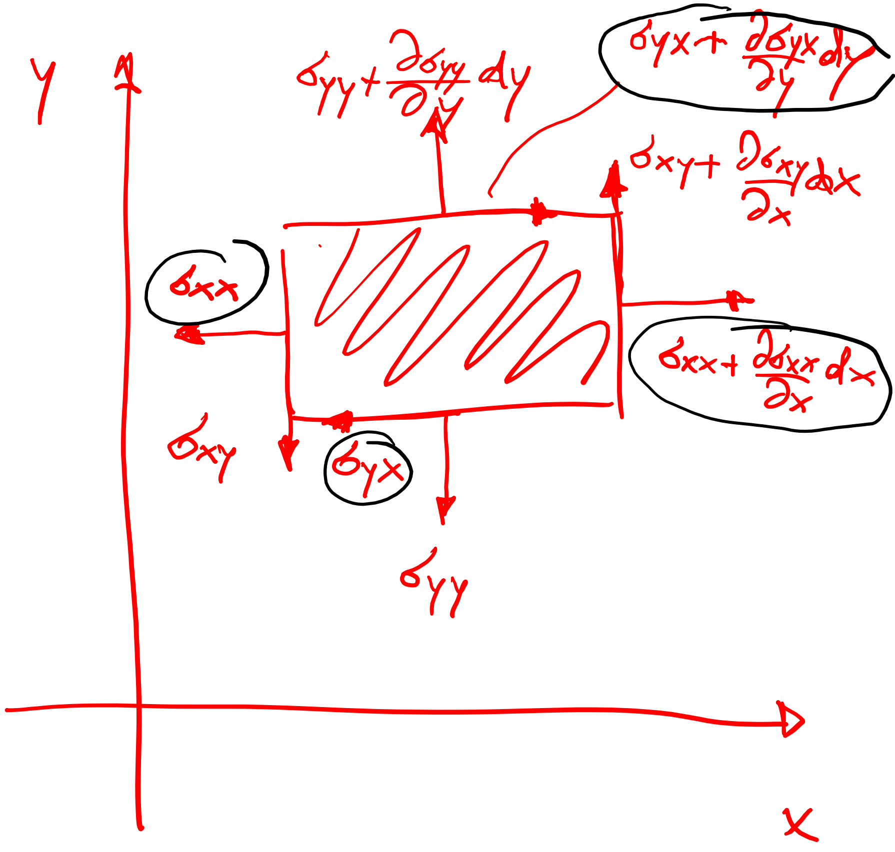
$$\sum F_x = F_{g,x} + F_{s,x}$$

$$F_{g,x} = M g_x = \rho\, dx\, dy\, g_x$$



$$F_{s,x} = -\cancel{\sigma_{xx}}\,dy - \cancel{\sigma_{yx}}\,dx + \left[\cancel{\sigma_{xx}} + \frac{\partial \sigma_{xx}}{\partial x}dx\right]dy + \left[\cancel{\sigma_{yx}} + \frac{\partial \sigma_{yx}}{\partial y}dy\right]dx$$

$$= \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y}\right)dx\,dy$$



$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\cancel{dx\,dy} = \left[\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y}\right]\cancel{dx\,dy}$$

$$\rho\left(\frac{\partial v}{\partial t} + \underbrace{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}}_{\bar{V}\cdot\nabla v}\right) = \rho g_y + \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}$$

$$(u\ \ v)\cdot\left(\frac{\partial v}{\partial x}\ \ \frac{\partial v}{\partial y}\right)$$

$$\sigma_{xx} = -p + \tau_{xx}$$

$$\sigma_{yy} = -p + \tau_{yy}$$

$$\sigma_{xy} = \tau_{xy}$$

$$\sigma_{yx} = \tau_{yx}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} =$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \rho g_y - \frac{\partial p}{\partial y} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} =$$

$$\boxed{\tau_{i,j} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

$$\tau_{xx} = \mu\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right) = 2\mu\frac{\partial u}{\partial x}$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{yy} = 2\mu\frac{\partial v}{\partial y}$$

$$\tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 2\mu \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial y}\right)$$

cont: $\boxed{\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}}$

$$= 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} - \mu \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

$$= \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\boxed{\rho\left(\frac{\partial u}{\partial t} + \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}}_{\bar{V} \cdot \nabla u}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu \underbrace{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)}_{\nabla^2 u}}$$

$$\rho\left(\frac{\partial u}{\partial t} + \bar{V} \cdot \nabla u\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu \nabla^2 u$$

y?

$$\boxed{\rho\left(\frac{\partial v}{\partial t} + \bar{V} \cdot \nabla v\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu \nabla^2 v}$$

$$\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$(u \;\; v) \cdot \left(\frac{\partial v}{\partial x} \;\; \frac{\partial v}{\partial y}\right) = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$