### **Topic 7. Heat exchangers**

### **Aim of this section**

Transfer of heat from one thermodynamic fluid to another, just like the transfer of money between trading parties, must be done at an exchange. Motivation for devising transfer of thermal energy in engineered systems arises in many common artefacts. Buildings require heating or cooling to maintain comfort for building occupants; incubators maintain a balanced environment for neonates; automobile engine coolant gathers high temperature thermal energy from the engine block and distributes it to the atmosphere via the radiator; utility boilers collect thermal energy from combustion gases, or nuclear reactor cores or geothermal etc sources and either raise steam directly or have a secondary heat transfer circuit to a further steam raising cycle; even lungs provide for heat transfer from the body. In all of these situations there are heat exchangers, whose duty is to maintain a barrier between the heat transfer fluids whilst providing an efficient conduit for the transfer of thermal energy from the hot stream to the cold. There exists a great deal of knowledge about heat exchangers and there are well established methods for analysis of common heat exchanger types, and this fundamentally important, commonplace and taken for granted aspect of thermodynamic systems deserves attention from the engineer.

**Aim**: identify the relationship between heat transfer and thermal capacity and calculate the heat exchanger size or capacity using heat exchanger analysis methods.

**Objectives**: Identify a regenerator and a recuperator; identify temperature profile in a tube in tube heat exchanger; relate heat transfer rate to thermal energy transfer; identify the logarithmic mean temperature difference (LMTD) of a heat exchanger based on desired temperature changes of the two fluid streams as a useful indicator of the average temperature difference between the fluids over the entire heat exchanger surface; calculate size of heat exchanger area based on LMTD; identify counter current and co-current configuration of the fluids in a tube in tube heat exchanger; calculate heat transfer rate in a heat exchanger using LMTD; method of using F correction factor from charts for more complicated heat exchanger geometries; understand and use capacity rate, C, of a flowing fluid in calculation of thermal transfer rate; identify the effectiveness-number of transfer units ( $\varepsilon$ -NTU) relationship of a heat exchanger; calculate the maximum possible heat transfer rate of a heat exchanger; use formulae or charts for  $\varepsilon$ -NTU calculations; understand the implications of fouling due to continuous flow of real fluids.

## **1. Definition of recuperator and regenerator**

Heat exchangers provide a method for exchanging thermal energy between a hot fluid and a cold fluid purely for the purpose of taking the thermal energy from a place where it can be easily drawn into the heat exchanger and where it is not in a useful form for a particular process to deliver it to another fluid which cycles around the process in a useful way. Heat exchangers are firstly divided into the category of **recuperator** which describes the most familiar heat exchangers where one fluid passes a wall separating it physically from the other fluid flow allowing heat transfer from the hot to the cold fluid through the wall and **regenerator** which describes the less familiar type of heat exchanger where a conductive thermal mass (such as a block of metal) absorbs the thermal energy from the hot source to deliver it at another time, or in another position after transfer, into the cold fluid. A good example of the regenerator is found on the air inlet of a combustion power generation



**Figure 1 schematic of a regenerator used in a power station air inlet exchanging heat with the exhaust gases. <http://www.pias-usa.com/products/utility/airheaters.html>**

station and is illustrated in the figure below.

The regenerator enables recovery of some of the energy from the power station

exhaust gases. A product is illustrated on the right side of Figure 1, the radial rods are arranged in a rotor, and the central seal has one pipe attached on the hot gas side and a separate pipe on the air side – the rods are rotated from one side to the other and pick up heat on the hot side and give it up on the cold side.



The recuperator in its simplest form is a tube in a tube as shown in Figure 2. A central pipe has a hot fluid and an outer pipe has a cold fluid. The fluids exchange heat across the wall that

**Figure 1 schematic of the simplest recuperator.**

separates them.

The remainder of this topic will be concerned with recuperator heat exchangers only.

**Example:** Calculate the rate of heat lost from a domestic boiler producing combustion gases at 140°C with a specific heat capacity  $c_p = 1.2$  kJ/kgK and at a rate of 3 g/s. The ambient temperature is  $15^{\circ}$ C.

## **2. Temperature profile in a recuperative heat exchanger – tube in tube.**

The tube in tube type of heat exchanger is called **shell and tube** heat exchanger – the outer tube being the shell. Given this simple arrangement the flow streams can flow either in the same direction, in which case it is a **parallel flow heat exchanger** or in the opposite direction, in which case it is a **counter flow heat exchanger**. The temperature of the hot fluid will become less and the temperature of the cold fluid will increase as they progress from one end to the other of the shell and tube heat exchanger.



Figure 3 shows a schematic diagram of a single shell and tube heat exchanger. Hot fluid flows in the innertube from left to right, and cool fluid flows in the cylindrical shell from right to left – a counter flow situation. The shell has

end plates to contain the fluid within it around the inner pipe, and a connecting pipe at either end. **Figure 3 schematic single shell and tube, with counter flow.**

Given a particular heat transfer coefficient on the interface wall for each fluid, the rate of heat transfer will be determined by the temperature of the hot and cold streams at any particular point along the length of the heat exchanger, and the temperature change from one end to the other of each fluid will depend on their flow rate and heat capacity. For hot oil flowing in the tube and water in the shell, a calculation can be performed as outlined below in an iterative manner to show the temperature distribution along the length.



**Figure 4 counter flow of oil and water in a single shell and tube**

The chart shows that the oil entering at the left hand end cools as it encounters the water. The oil mass rate and specific heat capacity mean that it has a **capacity rate, C** of 0.012 kW/K, (oil is subscript 1 in the table and water subscript 2). This capacity rate is less than that of water which has 0.042 kW/K and therefore the slope of the oil cooling is steeper than that of the water heating up.

If the shell direction of flow were reversed, so the arrows on the shell pipes on the figure pointed down instead of up, the it would be parallel flow. The situation with the same flow rates would result in a different temperature profile as illustrated in Figure 5, where it can be seen that the oil is cooled down faster, and that the exit temperature of the water is less.



**Figure 5 parallel flow of oil and water in a single shell and tube.**

# **3. Heat transfer calculation method for single shell and tube**

The purpose of the heat exchanger is to transfer thermal energy from one fluid to the other, and the calculation is down to heat transfer, which is convective. There are correlations for convective heat transfer as described in the convective heat transfer lecture. For the purpose of the heat exchangers here, it is sufficient to say that there will be an overall heat transfer coefficient formed not only by the local conditions at a particular point in the heat exchanger and for the convection for each fluid and the heat exchanger wall material, but for the entire heat exchanger allowing for fluid property changes due to temperature variation. The calculation is beyond the scope of this course, but it is useful to identify the techniques used to demonstrate they are within reach by application of heat transfer.



For the elemental length of the single shell and tube heat exchanger illustrated in Figure 6, the area of the pipe over the elemental length is dA, and there will be an associated heat transfer rate

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dq, dependent on the local fluid temperature difference and the thermal resistance between them.

Heat transfer tells us, that at any position in the tube, the heat transfer rate between the two fluids will be:

$$
\dot{Q} = \frac{\Delta T_{overall}}{R_{th}} = \frac{\Delta T_{A-B}}{\frac{1}{h_i 2\pi r_i} + \frac{r_o}{2\pi kL} + \frac{1}{h_o 2\pi r_o}}
$$

Where the log term due to pipe conduction is usually negligibly small – so it reduces to two convective terms. Or for a more straightforward analysis in the context of heat exchangers, assuming that the thermal resistances have been calculated and we have an overall heat transfer coefficient, U:

$$
\dot{Q} = U A \Delta T_{A-B} = U A (T_A - T_B)
$$

at any position in the concentric tubes system for an elemental length, and:

$$
UA = \frac{1}{\sum R_{th}}
$$

This is the rate of heat transfer across the surface which separates the two fluids at a particular position along the length. The fluids flowing in opposite directions to each other in this case, and exchange heat at this rate exchanging thermal energy, so that what the hot stream loses, the cold stream gains:

$$
\dot{Q} = \{ \dot{m}c_p (T_{\text{in}} - T_{\text{out}}) \}_{\text{hot}} = \{ \dot{m}c_p (T_{\text{out}} - T_{\text{in}}) \}_{\text{cold}}
$$

Therefore the rate of cooling can be calculated and the temperature profile of the heat exchanger with these fluids is established. This is what was done to produce the temperature profiles in the previous section.

This is an iterative approach due to the elemental nature of the analysis. For the heat exchanger calculation when the overall heat transfer coefficient is known, it would be useful to have a representative overall temperature difference.

**Example:** A tube and shell heat exchanger has tube internal radius 6 mm and a wall thickness of 1mm. The inner heat transfer coefficient is 1500 W $\cdot$ m<sup>-2</sup>K<sup>-1</sup> and the outer surface is 2000 W $\cdot$ m<sup>-2</sup>K<sup>-1</sup>. The thermal conductivity of the pipe wall is 100 W $\cdot$ m<sup>-2</sup>K $\cdot$ <sup>1</sup>. Calculate thermal resistance and hence overall heat transfer coefficient.

# **4. Logarithmic mean temperature difference.**

Since the temperature difference varies along the flow path of the heat exchanger, it will be useful to have an overall representative temperature difference which is an accurate representation. This is found in the LMTD, which is a mean temperature difference, and often represented as  $\Delta T_m$ .

# **Derivation**

It can be derived as follows, and this is shown purely to demonstrate how it is derived.

Heat transferred between hot, h, and cold, c is:

$$
dq = (T_h - T_c)U_A dA
$$

From capacity of each fluid, heat absorbed (or lost) is:

$$
d(T_h - T_c) = dT_h - dT_c = -\frac{dq}{C_h} - \frac{dq}{C_c}
$$

Combine equations to eliminate q:

$$
\frac{d(T_h - T_c)}{-\left(\frac{1}{C_h} + \frac{1}{C_c}\right)} = (T_h - T_c)U_A dA
$$

Shuffle and integrate:

$$
ln \frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})} = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right)U_A A
$$
  
\n
$$
T_{h2} - T_{c2} = \Delta T_2
$$
  
\n
$$
T_{h1} - T_{c1} = \Delta T_1
$$

Also,

$$
\frac{1}{C_h} + \frac{1}{C_c} = \frac{1}{q} \left( \Delta T_1 - \Delta T_2 \right)
$$

So

$$
q = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} UA = \Delta T_m UA
$$

It is important to understand what the temperature differences refer to.  $\Delta T_1$  is the difference in temperature between the two fluids at one end, and  $\Delta T_2$  is the difference between their temperatures at the other end.

Example: Calculate the LMTD for a heat exchanger with hot fluid initial temperature 100°C and final 50°C and cool fluid initial temperature 20°C and final 40°C, for firstly the counter current case and then for the parallel flow case..

#### Calculating heat exchanger size

The LMTD approach can be used for calculating the surface area of heat transfer required in the heat exchanger.

$$
A = \frac{Q}{U \Delta T_m}
$$

**Example:** for the counter flow and parallel flow heat exchangers above, determine the surface area required to exchange heat at the rate of 1 kW; the overall heat transfer coefficient, *U*, is 120 W/m2K.

# **5. Using LMTD for calculating temperatures**

LMTD method requires the inlet and outlet temperatures. Therefore if these are not known then an iterative approach must be used:

- a) Guess outlet temperatures
- b) Calculate LMTD
- c) Calculate  $\dot{q}$
- d) Calculate outlet temperature for step a)

# **Comparison of parallel and counter flow**

By comparison of the calculated surface areas above, the counter flow heat exchanger will result in a smaller heat exchanger surface. By comparison of the parallel flow heat exchanger graph plotted in section 2, it can be seen that the temperature midway between the two streams will be lower for the separating wall temperature in the case of the parallel flow case, which may be important when the fluids approach the melting point of the heat exchanger material.

# **When the shape is more complicated**

The case of the single shell and tube is simple to analyse and can be attempted using 2<sup>nd</sup> year undergraduate heat transfer techniques. But for more complex designs the analysis relies on experimental data which has been accumulated by other researchers.



**Figure** *showing the Coates building boiler room, which receives high pressure hot water from the central heating plant of the university (underneath the big chimney on University Park campus), has large multipass shell and tube heat exchangers. One of them is illustrated here during maintenance, together with an inset figure from Bejan's book on Heat Transfer<sup>1</sup> , which demonstrates how the flows interact.*

<sup>&</sup>lt;sup>1</sup> Adrian Bejan, 1993, Heat Transfer, Wiley and Sons Inc



In a multipass heat exchanger the heat exchange is complicated. We still have a known U overall, but the temperature variation is complicated. Therefore use a correction factor from a chart, an example of which is shown below for three configurations of heat exchanger. Refer to 'Compact heat exchangers' by Kays and London<sup>2</sup>.

The effects of crossflow and multipass are accounted for by introducing a factor F such that:

$$
Q = UAF\Delta T_m
$$

The factor F is derived from calculations similar to the derivation of LMTD or read from graphs. The correction factor allows us to use the LMTD type of calculation for more complex geometries. Two new variables are introduced:

$$
P = \frac{t_2 - t_1}{T_1 - t_1}
$$

and

$$
R = \frac{T_1 - T_2}{t_2 - t_1}
$$

The meaning of these is apparent from the cases illustrated on the left. R is the ratio of the temperature changes in each fluid and hence of the thermal capacities. P is the ratio of temp change in one fluid to the maximum temp change available.

Read the correction factor from charts, knowing P on the horizontal axis, a selection of curves for various values of R, and corresponding correction factor, F from the vertical axis.

<sup>2</sup> WM Kays and AL London, 1998, Compact Heat Exchangers, Krieger Publishing.

Three cases shown from a book by Bowman, Mueller and Nagle in Heat Transfer, Bejan, 1993.

**Example:** calculate the geometrical correction factor for the previous example LMTD calculation for the 3 heat exchangers shown.

The idea of increasing complexity is to increase compactness. As a rule of thumb, a compact heat exchanger is one that has a heat exchange surface >700 $m^2$  per  $m^3$  of volume. Lungs are excellent – the passages are typically 0.1mm diameter, and they have approximately 30,000 m<sup>2</sup> per m<sup>3</sup>.

If P $\rightarrow$ 0, the stream having temperatures t<sub>1</sub> and t<sub>2</sub> has change of phase, i.e. if pressure drop is not too great  $t_1 \rightarrow t_2$ .

If R $\rightarrow$ 0, the stream having temperatures T<sub>1</sub> and T<sub>2</sub> has a change of phase.

If a stream has a phase change, the capacity rate is effectively infinite, because the fluid will absorb heat without changing temperature. These are the two limits on the charts.

## **6. The effectiveness-number of transfer units method**

An alternative method which examines the thermal conductance, UA, as well as the capacity rates, Chot and Ccold, this method introduces two dimensionless groups:

The Number of heat Transfer Units:

$$
NTU = \frac{UA}{(\dot{m}C_p)_{\text{min}}}
$$

where  $(mC_p)_{min}$  is the smaller capacity rate.

And the effectiveness:

$$
\varepsilon = \frac{\text{actual heat transfer rate}}{\text{maximum heat transfer rate}} = \frac{\dot{q}}{\dot{q}_{\text{max}}}
$$

$$
\dot{q}_{\text{max}} = C_{\text{min}} \Delta T_{\text{max}}
$$

This can best be considered in graphical terms; it asks the question if the UA could be increased to whatever we wanted, what is the maximum achievable heat exchange between the two fluids?



As the size of the exchanger increases, the fluid with the smaller capacity rate, which has the steeper gradient, in this case  $C_{cold}$ , exchanges heat with the hot fluid until the limit when the size is such that it leaves at the temperature of the hot fluid. This is the limit of the heat transfer. The minimum fluid may be either hot or cold fluid.

**Example:** Oil flows at the rate of 5 g/s and has a specific heat capacity of 2.4 kJ/kgK and water flows in the same heat exchanger at the rate of 10 g/s with a specific heat capacity of 4.18 kJ/kgK. Calculate the capacity rate of each fluid and determine which is the minimum capacity rate. Given that the hot oil enters at 140°C and the water at 20°C, calculate the  $\dot{q}_{max}$ .

### Calculations for NTU and

Usually calculations entail evaluating from knowledge of C and NTU values, and then evaluating q<sub>actual</sub>. No iteration is required. Expressions for **NTU** in terms of  $\varepsilon$  can be derived by analysis of the rearranged **LMTD** heat transfer equation:

$$
LMTD = \Delta T_m = \frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a / \Delta T_b)}
$$

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in terms of effectiveness and NTU by manipulation of the three equations. For example, a parallel flow gives the formula:

$$
NTU = \frac{ln \left[1 - \varepsilon \left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}}
$$

Which can be manipulated to give the effectiveness:

$$
\varepsilon = \frac{1 - exp \left[ -NTU \left( 1 + \frac{C_{min}}{C_{max}} \right) \right]}{1 + \frac{C_{min}}{C_{max}}}
$$

A formula is required for each configuration of heat exchanger, since the simple case of the parallel flow single shell and tube is similarly complicated as in the case of the LMTD when the geometrical arrangement is complicated. Fortunately, rather than using the formulae, data is presented in charts that can be read. If  $C_{min}/C_{max}$  =1, the capacity rates are balanced – balanced heat exchanger. If  $C_{min}/C_{max}$  =0, one stream,  $C_{max}$ , has phase change at nearly constant pressure. For the same geometrical arrangements as presented for the LMTD case, the same are presented for this method on the following page.

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**Example:** Calculate the number of transfer units (NTU) for the previous example for a heat exchanger with an overall heat transfer coefficient of 120 W/m<sup>2</sup>K and a surface area between the two fluids of 0.3  $m<sup>2</sup>$ .

**Example:** For each of the 3 heat exchangers in the figure on the left, calculate the effectiveness and hence the actual heat transfer rate for the case above.

## **7. Pressure drop and fouling**

## Work of pump or fan to drive the fluids

Calculation of the pressure drop depends on detailed knowledge of the path that the fluid follows in flowing through the heat exchanger. Expansions, entries, bends all need to be included. There is a pump factor that needs to be taken into account. For a liquid it is:

$$
\dot{W} = \frac{1}{\eta_p} \frac{\dot{m}}{\rho} (p_{\text{in}} - p_{\text{out}})
$$

Where  $\eta_p$  is the pump efficiency. The pressure difference depends upon the friction factor for the tubes and on the minor losses in bends, expansions and contractions.

For gases the work is:

$$
\dot{W} = \frac{1}{\eta_c} m c_p T_{\rm in} \left[ \left( \frac{p_{\rm out}}{p_{\rm in}} \right)^{R/c_p} - 1 \right]
$$

Where  $\eta_{\rm p}$  is the compressor efficiency.

Intensification of heat transfer is usually accompanied by an increase in pressure drop. This is because to increase heat transfer, more surface area, and more turbulent mixing are to be encouraged, both of which add friction to the flow.

## Fouling of the surfaces by contaminants over time

Over time the boundary surfaces corrode and acquire a scale coat, whilst the fluids gain impurities, so a fouling factor, r, is used for both shell and tube giving a revised overall heat transfer coefficient:

$$
\frac{1}{U_S} = \left[\frac{1}{h_S} + \frac{1}{h_T} \frac{A_S}{A_T}\right] + \left[r_S + r_T \frac{A_S}{A_T}\right]
$$

Where subscript s is shell-side, subscript t is tube-side.

The table gives representative Fouling Factors [m<sup>2</sup>K/W] related to various fluid flows and illustrates the conditions that might lead to fouling (from Chenoweth, 1990, in Heat Transfer, Bejan).



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**Example:** calculate the overall heat transfer coefficient change due to a heat exchanger with river water in a tube flowing at 1 m/s and steam without oil in the shell, given the heat exchanger area is a tube that is 5 m long and 38 mm diameter and is the same for shell and tube.