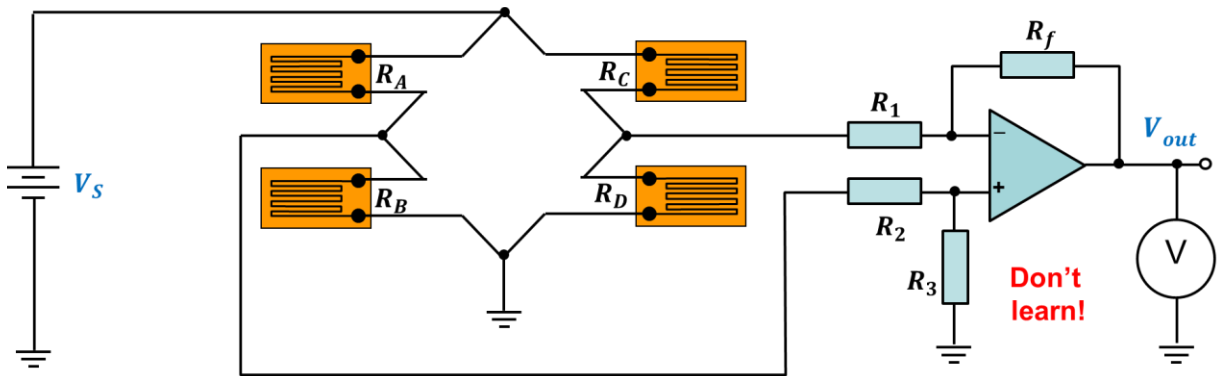


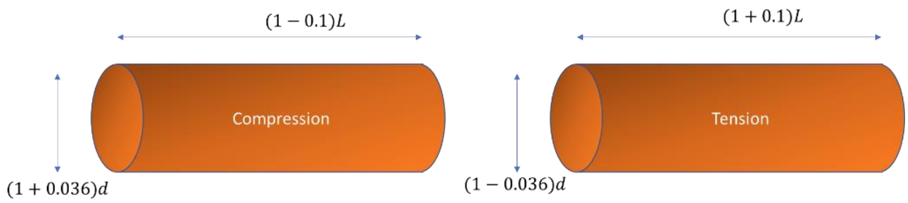
Lets say the strain gauge causes a compression (and tension) of 10% strain. This means the length was decreased (or increased, in case of tension) by 10%.

Resistivity of Copper = $1.724 \times 10^{-8}\Omega m$

Poisson's Ratio of Copper = $0.36 = \frac{\delta\varepsilon_{trans}}{\delta\varepsilon_{axial}} = \frac{\delta\varepsilon_{trans}}{0.1}$

$$\delta\varepsilon_{trans} = 0.36 \times 0.1 = 0.036$$

This means, if the length of the conductor increased by 10% (or 0.1), the diameter decreased by 3.6% (or 0.036). Similarly, for compression.



Now how does that translate to change in resistance?

$$R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2} = \rho\frac{4L}{\pi D^2} = \frac{4\rho}{\pi}\frac{L}{D^2}$$

Lets solve for compression,

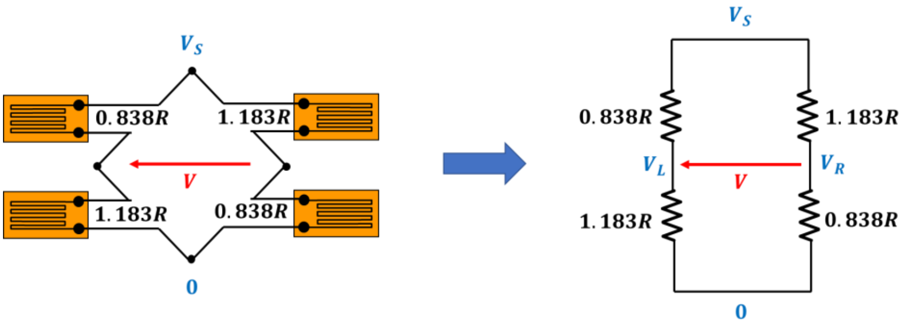
$$R_{comp} = \frac{4\rho}{\pi}\frac{0.9L}{(1.036D)^2} = \frac{4\rho}{\pi}\frac{0.9L}{1.0733D^2} = 0.838\frac{4\rho}{\pi}\frac{L}{D^2} = 0.838R$$

Similarly, for tension,

$$R_{tens} = \frac{4\rho}{\pi}\frac{1.1L}{(0.964D)^2} = \frac{4\rho}{\pi}\frac{1.1L}{0.9292D^2} = 1.183\frac{4\rho}{\pi}\frac{L}{D^2} = 1.183R$$

This means, the resistance dropped to 83.8% of original value (or, by 16.2%) under compression, and increased to 118.3% of original value (or, by 18.3%).

The Wheatstone Bridge would look something as follows:

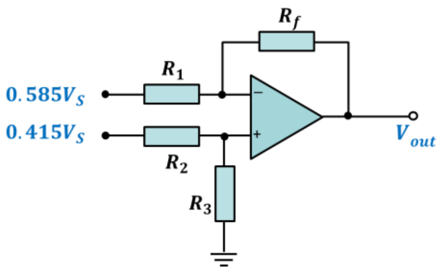


Using the familiar equation for a potentiometer,

$$V_L = V_s \frac{1.183R}{0.838R + 1.183R} = \frac{1.183R}{2.021R} = 0.585V_s$$

$$V_R = V_s \frac{0.838R}{0.838R + 1.183R} = \frac{0.838R}{2.021R} = 0.415V_s$$

So, a voltage is produced which can be amplified with the Op-Amp.



Now, we must solve the op amp circuit. Here are the equations we can start with:

1. $V_{out} = A(V_+ - V_-)$
2. $V_+ = \frac{R_3}{R_2+R_3} 0.415\ V_s$
3. $\frac{0.585\ V_s - V_-}{R_1} = \frac{V_- - V_{out}}{R_f}$

Solving equation (3) further to isolate V_-:

$$\frac{0.585\ V_s - V_-}{R_1} = \frac{V_- - V_{out}}{R_1}$$

$$\Rightarrow 0.585\ R_f V_s - R_f V_- = R_1 V_- - R_1 V_{out}$$

$$\Rightarrow 0.585\ R_f V_s + R_1 V_{out} = R_1 V_- + R_f V_-$$

$$\Rightarrow \frac{0.585\ R_f V_s + R_1 V_{out}}{R_1 + R_f} = V_-$$

Now, we can replace the Op-Amp input voltage values in the open-loop gain equation (1):

$$V_{out} = A(V_+ - V_-)$$

$$\Rightarrow V_{out} = A\left(\frac{R_3}{R_2 + R_3} 0.415\, V_s - \frac{0.585\, R_f V_s + R_1 V_{out}}{R_1 + R_f}\right)$$

$$\Rightarrow V_{out} + A\frac{R_1 V_{out}}{R_1 + R_f} = A\left(\frac{0.415\, R_3}{R_2 + R_3}\, V_s - \frac{0.585\, R_f V_s}{R_1 + R_f}\right)$$

$$\Rightarrow V_{out}\left(1 + A\frac{R_1}{R_1 + R_f}\right) = V_s\left(A\frac{0.415\, R_3}{R_2 + R_3} - A\frac{0.585\, R_f}{R_1 + R_f}\right)$$

Dividing both sides by A:

$$\Rightarrow V_{out}\left(\frac{1}{A} + \frac{R_1}{R_1 + R_f}\right) = V_s\left(\frac{0.415\, R_3}{R_2 + R_3} - \frac{0.585\, R_f}{R_1 + R_f}\right)$$

Since $A \gg 1$, we can easily replace $\frac{1}{A} \approx 0$:

$$\boldsymbol{\Rightarrow V_{out}\left(\frac{R_1}{R_1 + R_f}\right) = V_s\left(\frac{0.415\, R_3}{R_2 + R_3} - \frac{0.585\, R_f}{R_1 + R_f}\right)}$$

Above is the worked mathematics to solve a problem like this. As you see, it is simply Kirchhoff's Laws and simple maths! Let's produce a general equation using $\delta\varepsilon_{axial}$ (instead of 10%).

$$R_{comp} = \frac{4\rho}{\pi}\frac{(1 - \delta\varepsilon_{axial})L}{(1 + \upsilon\delta\varepsilon_{axial})^2 D^2} = \frac{(1 - \delta\varepsilon_{axial})}{(1 + \upsilon\delta\varepsilon_{axial})^2} R$$

$$R_{tens} = \frac{4\rho}{\pi}\frac{(1 + \delta\varepsilon_{axial})L}{(1 - \upsilon\delta\varepsilon_{axial})^2 D^2} = \frac{(1 + \delta\varepsilon_{axial})}{(1 - \upsilon\delta\varepsilon_{axial})^2} R$$

Then, we find the voltage produced across the Wheatstone Bridge as a result:

$$V_L = V_s \frac{\frac{(1 + \delta\varepsilon_{axial})}{(1 - \upsilon\delta\varepsilon_{axial})^2} R}{\frac{(1 - \delta\varepsilon_{axial})}{(1 + \upsilon\delta\varepsilon_{axial})^2} R + \frac{(1 + \delta\varepsilon_{axial})}{(1 - \upsilon\delta\varepsilon_{axial})^2} R}$$

$$\Rightarrow V_L = V_s \frac{\frac{(1 + \delta\varepsilon_{axial})(1 + \upsilon\delta\varepsilon_{axial})^2}{(1 - \upsilon\delta\varepsilon_{axial})^2 (1 + \upsilon\delta\varepsilon_{axial})^2}}{\frac{(1 - \delta\varepsilon_{axial})(1 - \upsilon\delta\varepsilon_{axial})^2 + (1 + \delta\varepsilon_{axial})(1 + \upsilon\delta\varepsilon_{axial})^2}{(1 + \upsilon\delta\varepsilon_{axial})^2 (1 - \upsilon\delta\varepsilon_{axial})^2}}$$

$$\boldsymbol{\Rightarrow V_L = V_s \frac{(1 + \delta\varepsilon_{axial})(1 + \upsilon\delta\varepsilon_{axial})^2}{(1 - \delta\varepsilon_{axial})(1 - \upsilon\delta\varepsilon_{axial})^2 + (1 + \delta\varepsilon_{axial})(1 + \upsilon\delta\varepsilon_{axial})^2}}$$

Similarly,

$$\boldsymbol{V_R = V_s \frac{(1 - \delta\varepsilon_{axial})(1 - \upsilon\delta\varepsilon_{axial})^2}{(1 - \delta\varepsilon_{axial})(1 - \upsilon\delta\varepsilon_{axial})^2 + (1 + \delta\varepsilon_{axial})(1 + \upsilon\delta\varepsilon_{axial})^2}}$$