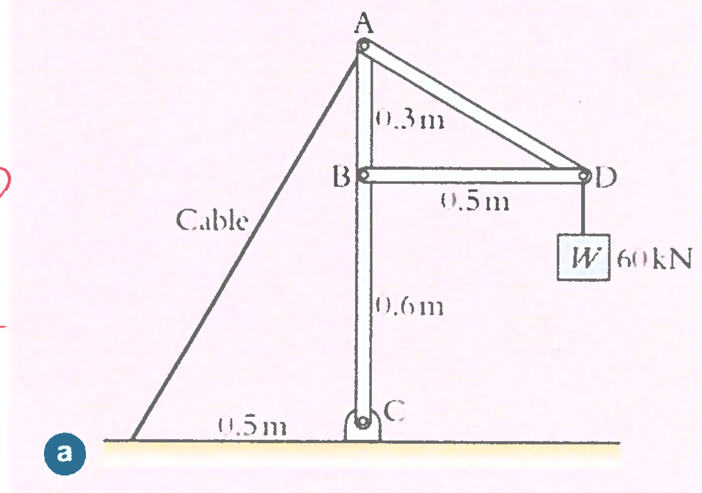
Worked example: pin-jointed structure

Question: the pin-jointed structure shown in the right supports a weight of 60 kN. Neglecting the weights of the members of the structure, use a graphical solution to determine the reaction force at point C and the tension in the cable.



Step 1: FBD

T/W/Rc must meet at a point

trigonometry

Three-force principle

$\tan\alpha = \frac{0.5\,m}{0.9\,m}$ $\alpha = \arctan\left(\frac{0.5}{0.9}\right)$

$\alpha = 29.05°$

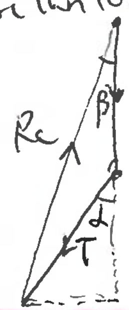
$\frac{0.3+0.6}{0.5} = \frac{L}{0.5+0.5}$

$L = 1.8$

$\tan\beta = \frac{0.5}{1.8}$

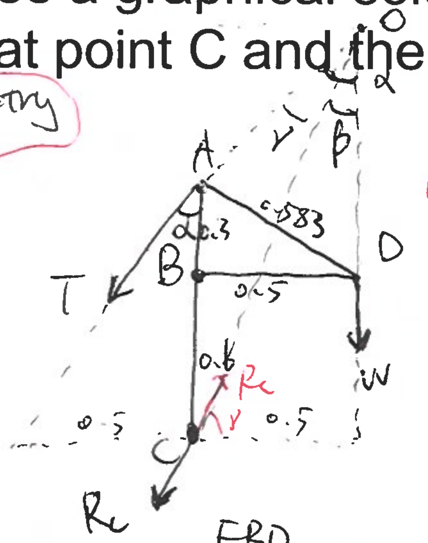
$\beta = \arctan\left(\frac{0.5}{1.8}\right)$

$\beta = 15.52°$

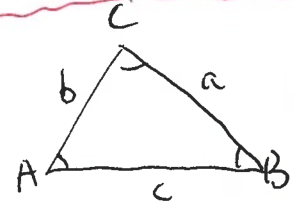


force polygon

(close the polygon)



law of sines   sine rule



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{R_c}{\sin(180-29.05)} = \frac{T}{\sin 15.52°} = \frac{W = 60\,kN}{\sin(180-15.52°-150.95°)} = 256.46$

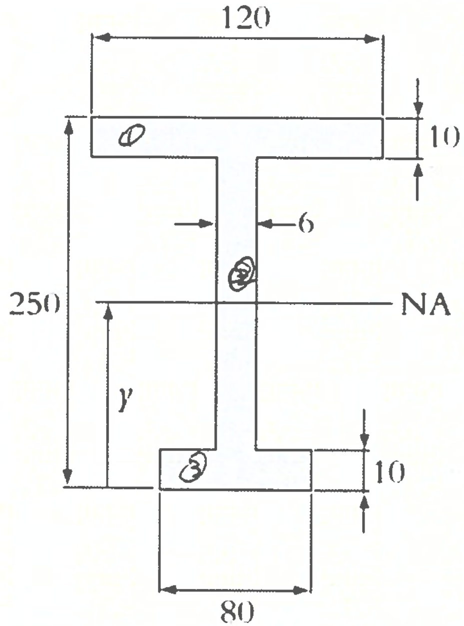
$13.53°$

$R_c = 256.46 \times \sin(180-29.05) = \underline{124.5\ kN}$

$T = 256.46 \times \sin 15.52° = \underline{68.6\ kN}$

Worked example: Uniformly distributed load – I-section

Question: The cross-section of an unsymmetric I-section beam and its dimensions are shown in the right. The beam is simply supported over a span of 5 metres and carries a uniformly distributed load of 5 kNm^{-1} over its full span. Assume that the beam is made of steel with E = 200 GPa. Calculate the maximum compressive and tensile stresses in the beam and the radius of curvature at the point of maximum bending moment.



"Unit 1 Solid mechanics" in "An Introduction to Mechanical Engineering: Part 1", by M. Clifford et al., Hodder Education.

$$M_{max} = \frac{wL^2}{8} = \frac{5\,kNm^{-1} \times (5\times5\,m^2)}{8} = 15.625\ kNm$$

Beam equation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} \qquad \frac{1}{R} = k$$

$$A_{tot} \cdot \bar{y} = \sum A_i \cdot y_i$$

$$A_{tot} = 120\times10 + 6\times(250-10-10) + 10\times80 = 3380\ mm^2$$

$$A_1 = 1200,\quad A_2 = 6\times230 = 1380,\quad A_3 = 800$$

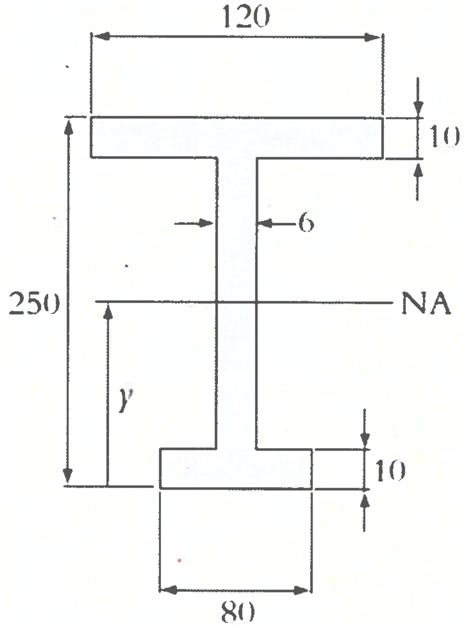
$$y_1 = 245,\quad y_2 = 125,\quad y_3 = 5$$

$$\rightarrow 3380\times\bar{y} = 1200\times245 + 1380\times125 + 800\times5$$

$$\rightarrow \bar{y} = 139.2\ mm$$ from the bottom surface

Worked example: Uniformly distributed load – I-section

subsection	I	A	$h = \lvert y_i - \bar{y} \rvert$	Ah^2	$I + Ah^2$
flange (h, NA)	$\frac{120 \times 10^3}{12} = 1.0 \times 10^4$	1200	$250 - 139.2 - 5 = 105.8$	1.343×10^7	1.344×10^7
web (h, NA)	$\frac{6 \times 230^3}{12} = 6.08 \times 10^6$	1380	14.2	0.278×10^6	6.36×10^6
(h, NA)	$\frac{80 \times 10^3}{12} = 6.67 \times 10^3$	800	134.2	1.44×10^7	1.441×10^7



"Unit 1 Solid mechanics" in "An Introduction to Mechanical Engineering: Part 1", by M. Clifford et al., Hodder Education.

Beam equation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} \qquad \frac{1}{R} = k$$

$$I_{NA} = 3.421 \times 10^7 \ mm^4 = 3.421 \times 10^{-5} \ m^4$$

$$1 Pa = 1 \frac{N}{m^2}$$

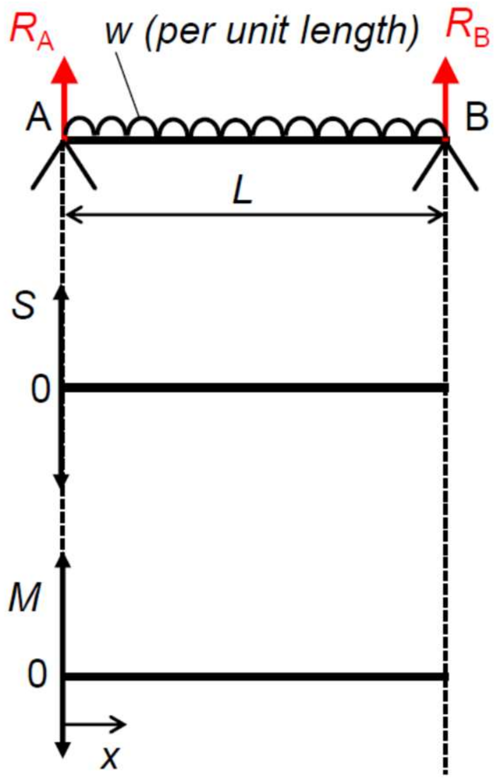
max tensile stress $\sigma_{bottom} = \frac{M_{max} y_{max}}{I} = \frac{15.625 \times 10^3 \ Nm \cdot 0.1392 \ m}{3.421 \times 10^{-5} \ m^4} = 63.58 \ MPa$

max compressive stress $\sigma_{top} = \frac{M_{max} y_{max}}{I} = \frac{15.625 \times 10^3 Nm \times (0.25 - 0.1392) m}{3.421 \times 10^{-5} \ m^4} = 50.61 \ MPa$

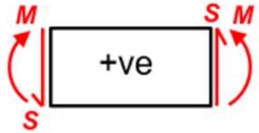
radius of curvature $\frac{M}{I} = \frac{E}{R} \Rightarrow R = \frac{E \cdot I}{M} = \frac{200 \times 10^9 Pa \cdot 3.421 \times 10^{-5} m^4}{15.625 \times 10^3 \ Nm} = 437.9 \ m$

Worked example: Uniformly distributed load – I-section

Maximum bending moment



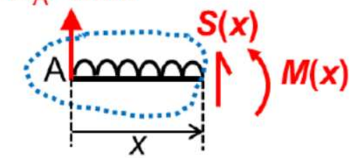
- Reactions



$$\uparrow \sum F = R_A + R_B - wL = 0 \qquad R_A + R_B = wL$$

$$\text{symmetry (or } \sum M) \qquad R_A = R_B = wL/2$$

- FBD of beam section



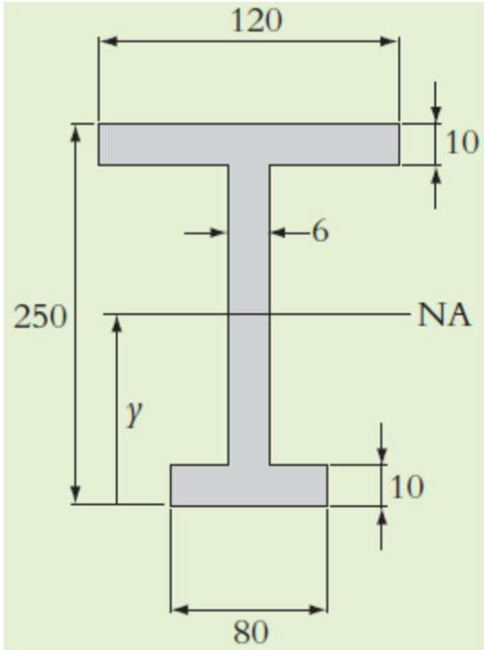
- Equilibrium equations

$$\uparrow \sum F = wL/2 - wx + S = 0$$

$$S = -wL/2 + wx$$

$$\circlearrowleft \sum M(x) = -wLx/2 + wx(x/2) + M = 0$$

$$M = wLx/2 - wx^2/2 \qquad \text{at } L/2, \; M = wL^2/8$$



"Unit 1 Solid mechanics" in "An Introduction to Mechanical Engineering: Part 1", by M. Clifford et al., Hodder Education.

The maximum bending moment occurs at the centre of the span (at L/2):

$$M_{max} = \frac{wL^2}{8} = \frac{5 \times 5^2}{8} = 15.625\ kNm$$

(slides from MMM1028 Statics and Dynamics, by Dr Davide De Focatiis)

The parallel axis theorem

Let us consider a generic shape with axis XX passing through its centroid.

The second moment of area about a different axis X'X' is given by

$$I_{X'X'} = \int_A (y+h)^2\, dA = \int_A y^2 dA + \int_A h^2 dA + \int_A 2yh\,dA$$

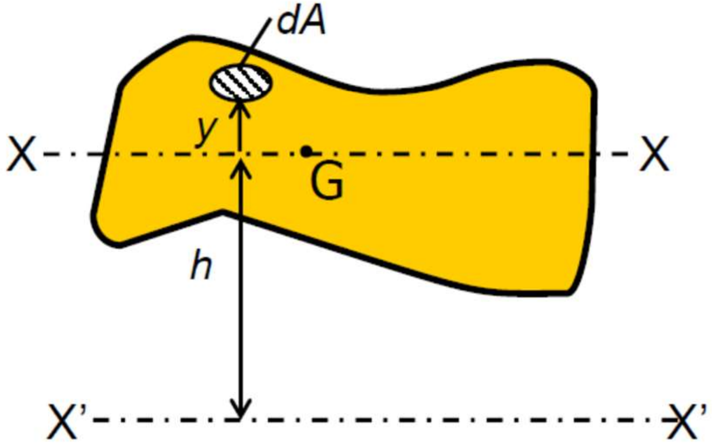
$$= \int_A y^2 dA + h^2 \int_A dA + 2h \int_A y\,dA$$

=0 (definition of the centroid)

$$\int_A y^2 dA = I_{XX} \qquad \int_A dA = A$$

in formula sheet!

$$\boxed{I_{X'X'} = I_{XX} + Ah^2}$$



(slides from MMM1028 Statics and Dynamics, by Dr Davide De Focatiis)