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Mechanics of Solids

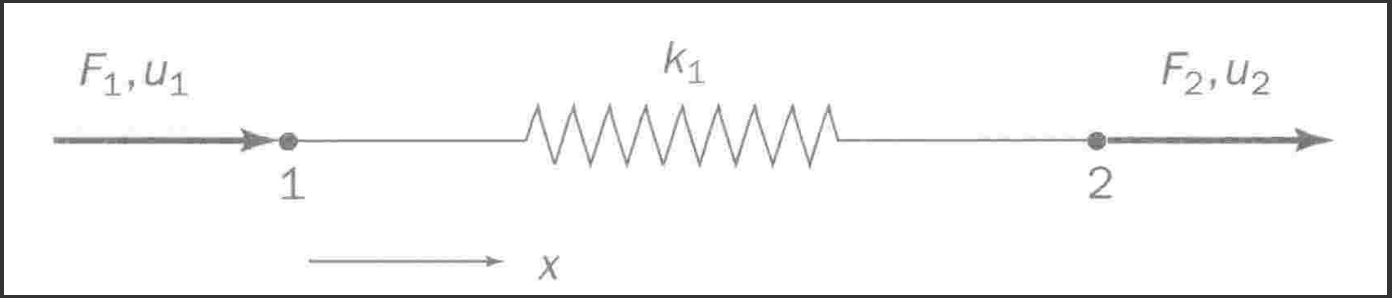
MMME2053

Finite Element Analysis

Lecture 2

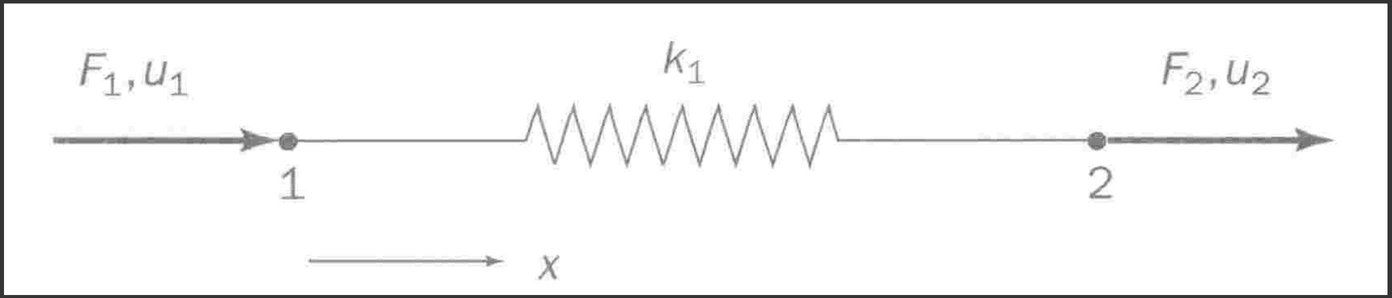
1D Spring Element

- Simplest element that we will use to build up understanding of how the Finite Element Method works



1D Spring Element

- The points of attachment to other parts or elements are called nodes (1 & 2). The nodal forces and displacements (*F* & *u* respectively) as well as the spring stiffness, k_1, are also shown. For a bar of length *L*, area *A* and elasticity modulus *E*, $k_1 = AE/L$

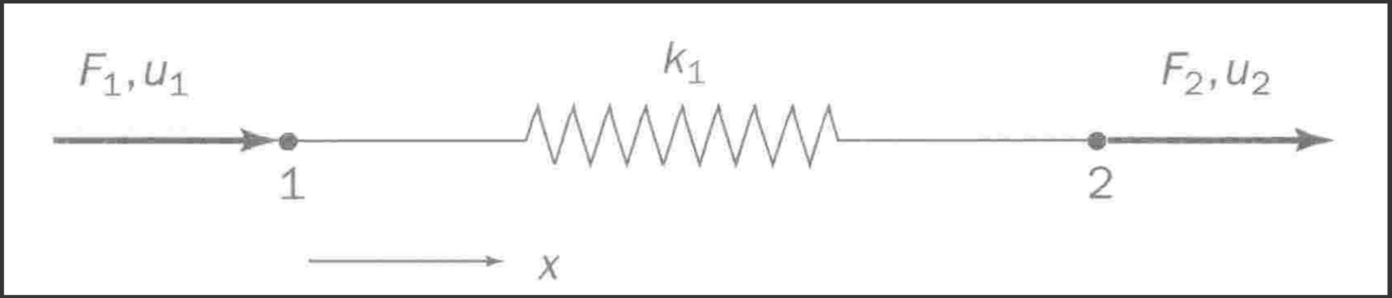


1D Spring Element

- The forces and displacements are related by the following equations:

$$F_1 = k_1(u_1 - u_2) = k_1u_1 - k_1u_2$$

$$F_2 = k_1(u_2 - u_1) = -k_1u_1 + k_1u_2$$



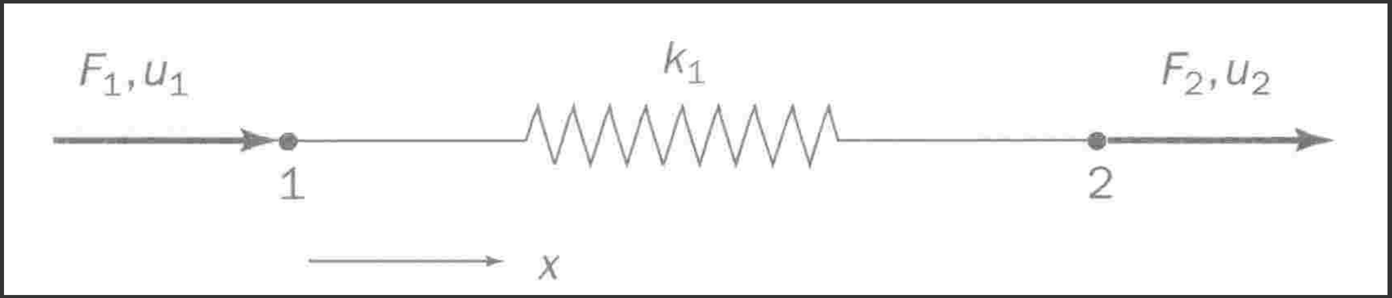
1D Spring Element

- Which can be expressed in matrix form as:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\{F\} = [K^e]\{u\}$$

- Where **[*K*ᵉ]** is the **Element** stiffness matrix

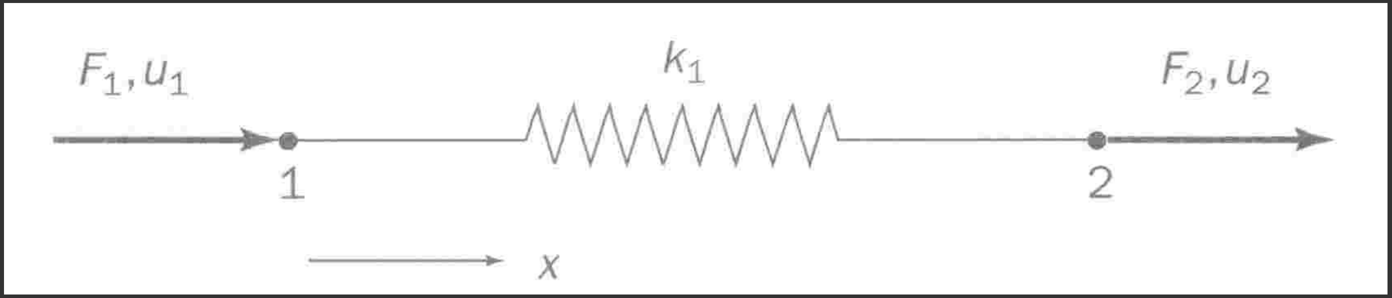


Vector quantities are denoted with braces { }, two dimensional arrays are contained in brackets [ ].

1D Spring Element

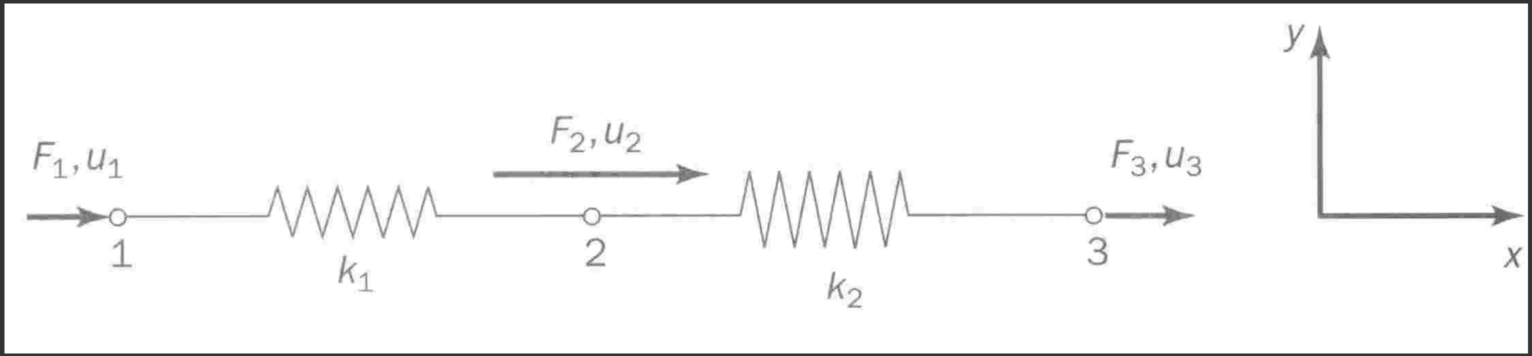
- Note that it is symmetrical. In general, whether for an element or a complete structure, the stiffness matrix is always symmetrical.

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$



Assembly of Spring Elements

- To perform analysis of real problems, we need to combine more than one element
- Shown below are two connected spring elements:



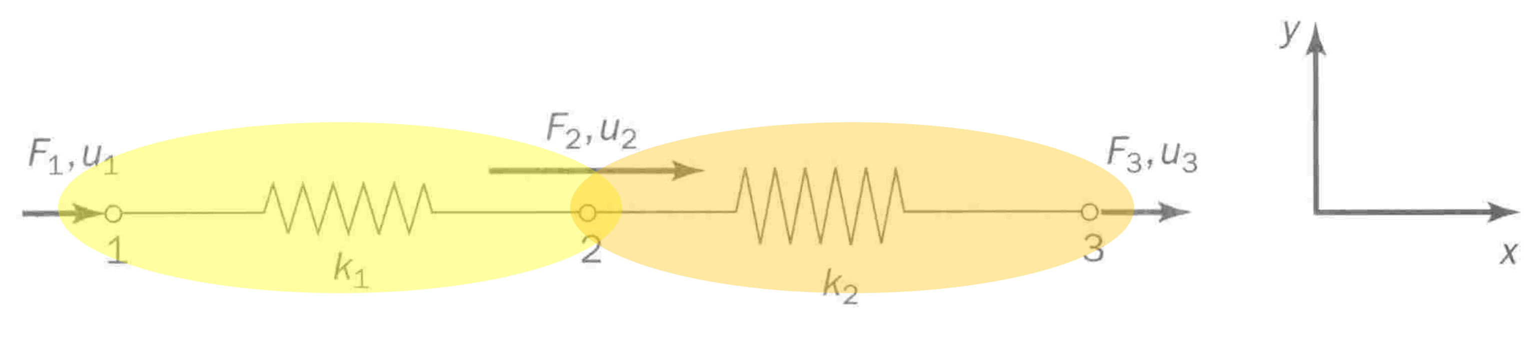
Assembly of Spring Elements

- For element 1:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

- And element 2:

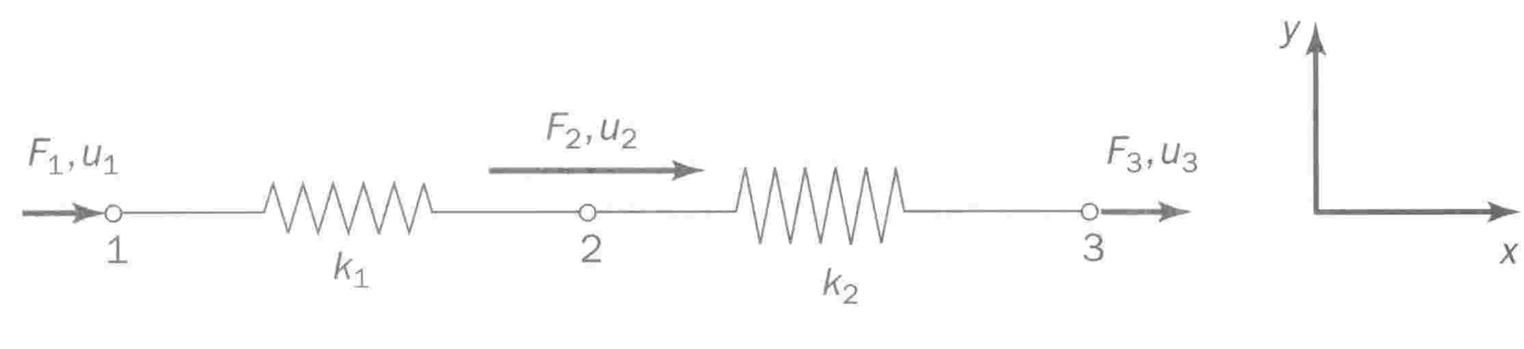
$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$



Assembly of Spring Elements

- If we expand the matrices to make them equivalent:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$



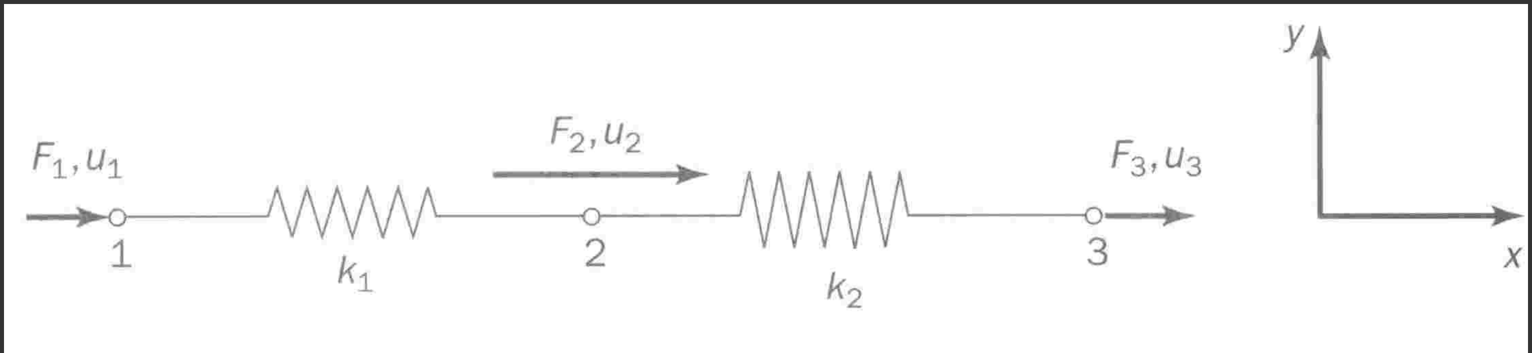
Assembly of Spring Elements

- We can then combine the matrices to form one stiffness matrix for the whole system:

e1

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

e2

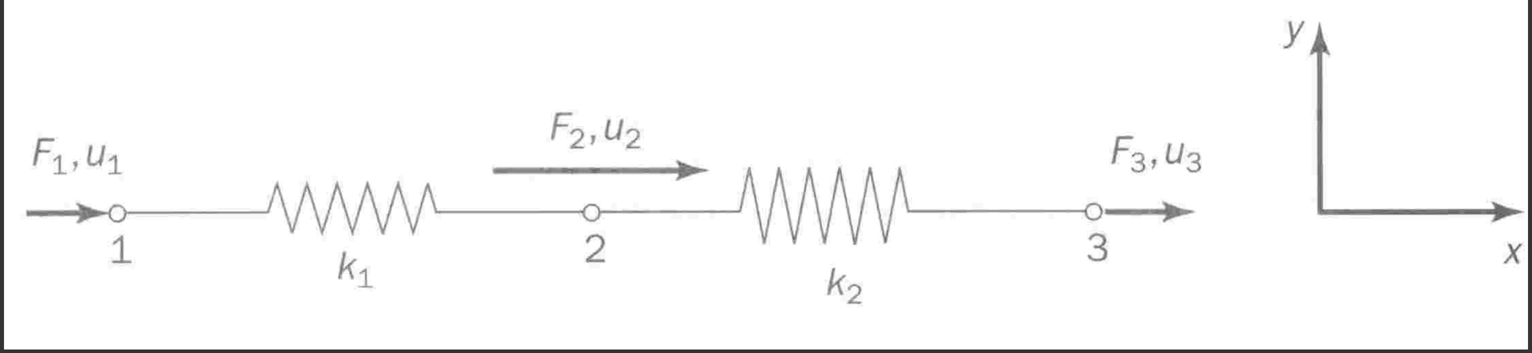


Assembly of Spring Elements

- Or:

$$\{F\} = [K]\{u\}$$

- Where $[K]$ is the **Global** stiffness matrix



Application of Spring Elements

- Now that we have a system of equations in matrix form that define the relationships between the forces and displacements in our system, we can start to use it to solve problems.

- If we subjects the system to a force(s) and sufficient boundary conditions are specified, the remaining forces and displacements can be found.

Learning Objectives

1. Recognise that FEA is a useful technique to aid the solution of many Structural Mechanics problems
2. Understand how 1D elements and the matrix method can be used to analyse uniaxial bars
3. Apply theory for 1D elements and the matrix method to an assembly of bars
4. Understand the derivation of the global stiffness matrix of a truss element