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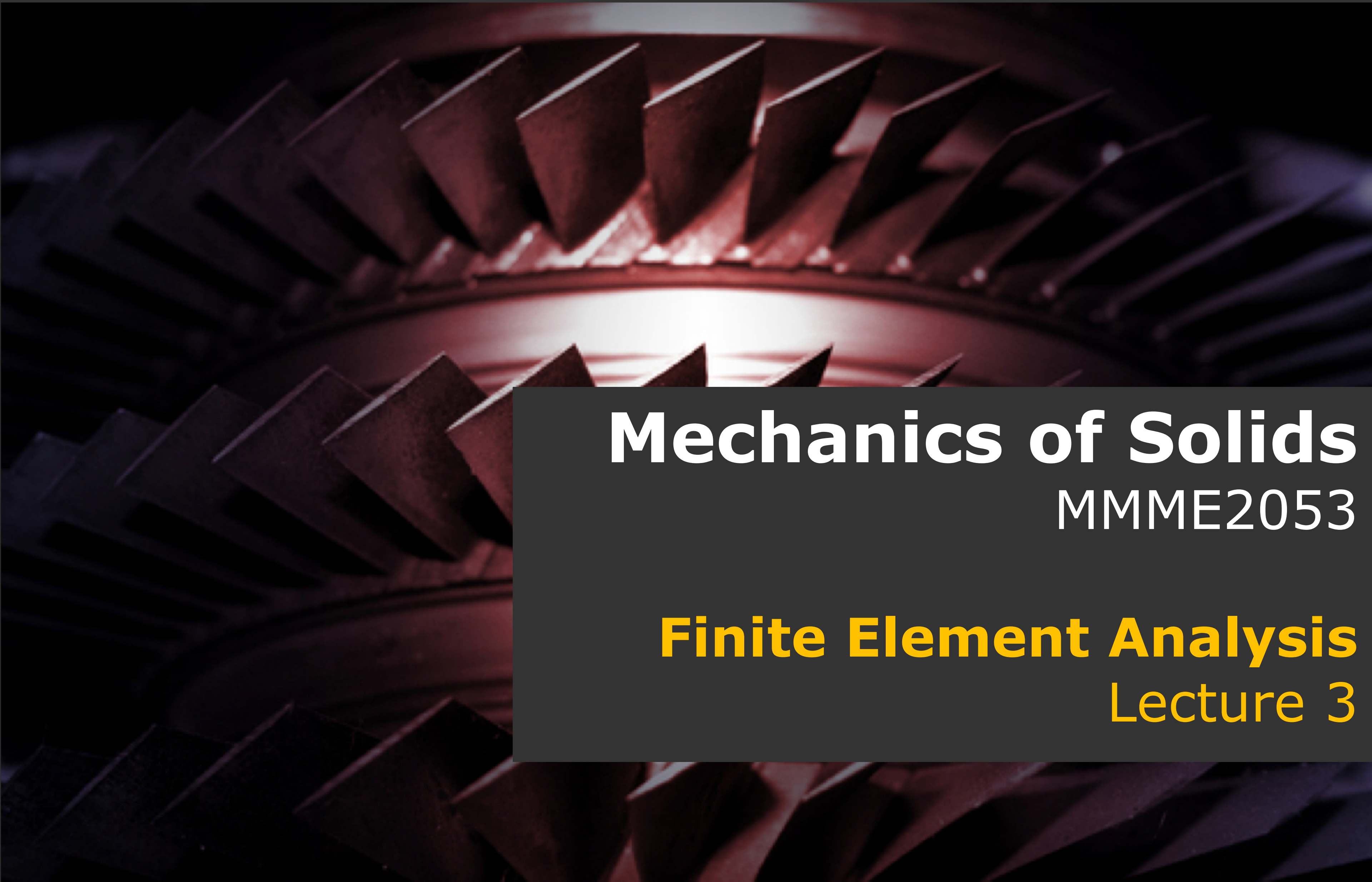


Mechanics of Solids

MMME2053

Finite Element Analysis

Lecture 3

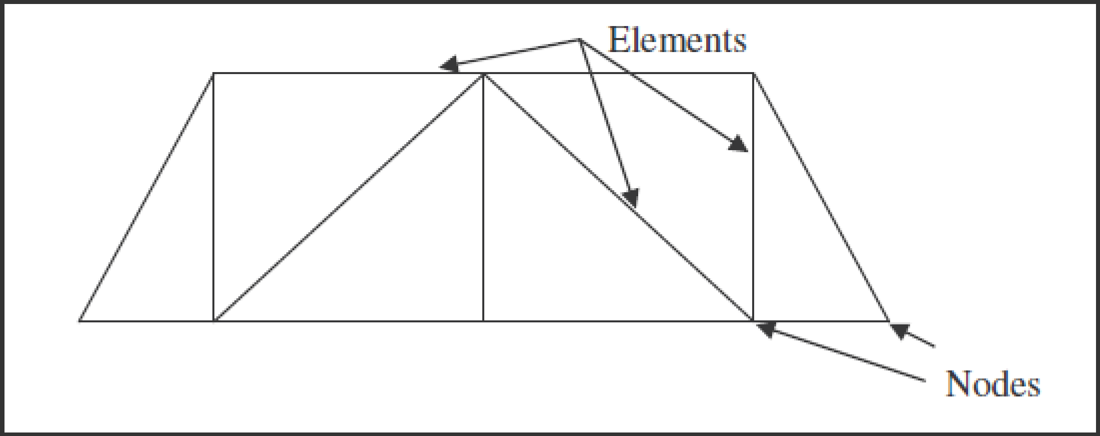
Truss Elements

- 1D spring elements are a good simple introduction to demonstrate the matrix method and the concept of stiffness matrices however, they have limited practical use.

- Truss elements are an extension of the spring element where each node has 2 degrees of freedom (DOFs) and they can be connected to form frame structures

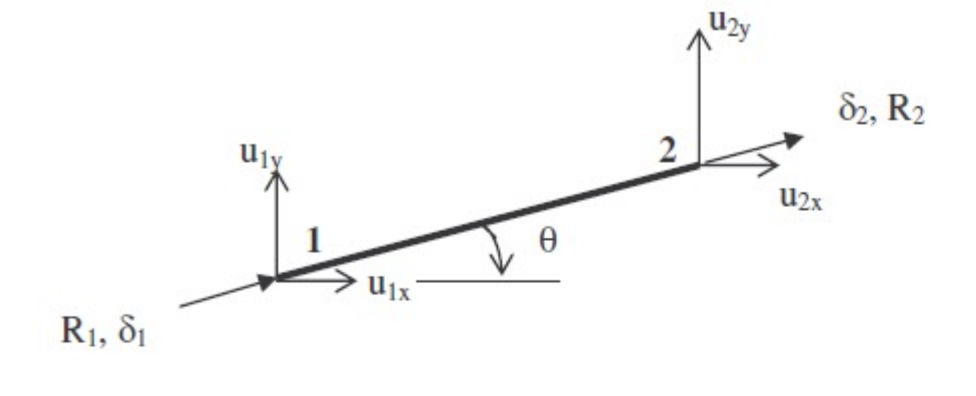
Truss Elements

- Truss element structure:



Truss Elements

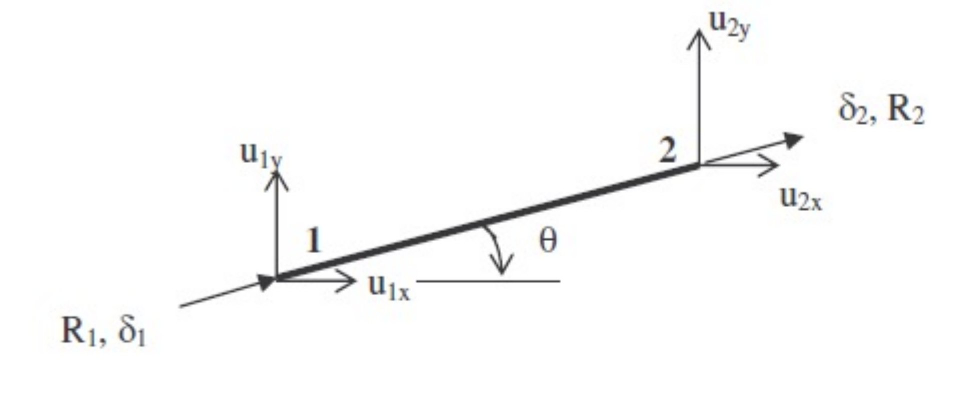
- The coordinate system for a truss element is shown here:



- We need to establish a relationship between the local deflections, δ_1 and δ_2, and the global deflections u_{x1}, u_{y1}, u_{x2} and u_{y2}

Truss Elements

- The coordinate system for a truss element is shown here:

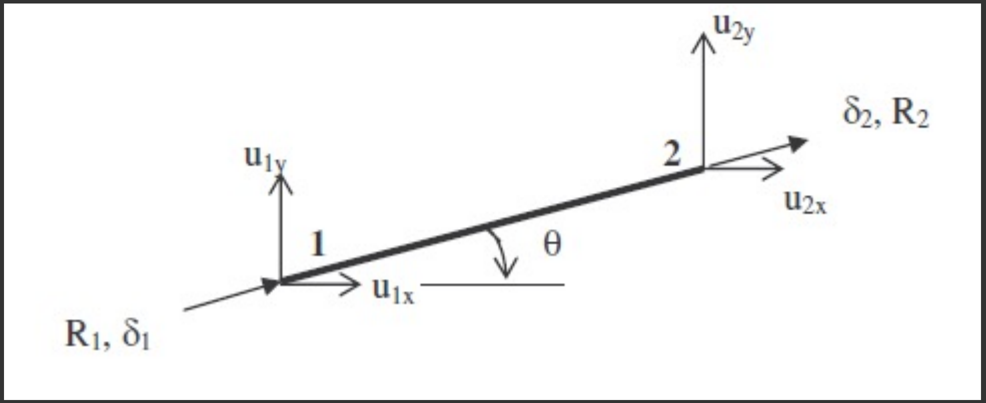


$$\delta_1 = U_{x1}\cos\theta + U_{y1}\sin\theta = cU_{x1} + sU_{y1}$$
$$\delta_2 = U_{x2}\cos\theta + U_{y2}\sin\theta = cU_{x2} + sU_{y2}$$

Where θ is the angle from the x-axis, c = cos θ and s = sin θ for brevity

Truss Elements

- In matrix form:



$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \end{Bmatrix}$$

$$\{\delta\} = [T]\{u\} \quad [1]$$

$[T]$ is a **Transformation** matrix

Truss Elements

- We also need to relate the local forces, R_1 and R_2 to the global forces in the same manner

- Work done is the same in local and global coordinate systems:

$$W = \{\delta_1 \quad \delta_2\} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \{u_{x1} \quad u_{y1} \quad u_{x2} \quad u_{y2}\} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

$$W = \{\delta\}^T \{R\} = \{u\}^T \{F\}$$

Truss Elements

- However:

$$\{\delta\} = [T]\{u\}$$

$$([T]\{u\})^T\{R\} = \{u\}^T\{F\}$$

$$[T]^T\{u\}^T\{R\} = \{u\}^T\{F\}$$

$$[T]^T\{R\} = \{F\} \qquad [2]$$

Truss Elements

- The local stiffness equation for the truss element is:

$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix}$$

- Recall that *k=AE/L* $\{R\} = [K^e]\{\delta\}$ [3]

Truss Elements

- Substituting [3] into [2]:

$$\{F\} = [T]^T [K^e]\{\delta\}$$

- Substituting in [1] for δ

$$\{F\} = [T]^T [K^e][T]\{u\}$$

Truss Elements

- Consider

$$[T][K^e][T] = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}$$

- Simplify

$$[K^e]_g = \left(\frac{AE}{L}\right) \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Truss Elements

$$[K^e]_g = \left(\frac{AE}{L}\right)\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

- Stiffness matrix for a truss element in global coordinates
- This element stiffness matrix is also symmetrical
- Matrix is 4 x 4 as each element has 4 unknown deflections (2 nodes with 2 DOFs)
- Again $c = \cos\theta$ and $s = \sin\theta$

13/14 FE Exam Question

- The pin-jointed framework is subjected to an external load as shown in Figure Q1b. If each member has a length *L*, cross sectional area *A* and modulus of elasticity *E:*
  - construct the stiffness matrix of the structure

    [10 marks]
  - determine expressions for the horizontal and vertical displacements at point B

    [3 marks]
  - determine expressions for the reaction forces at points A and C

    [5 marks]

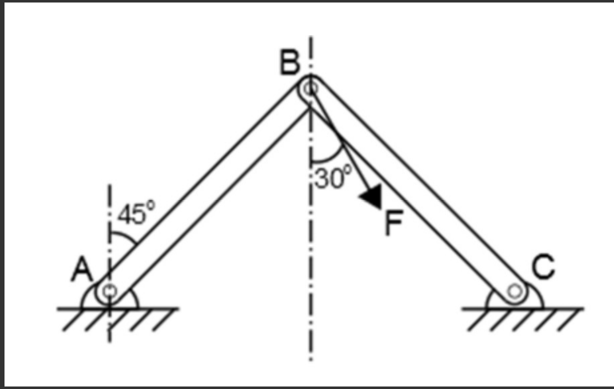
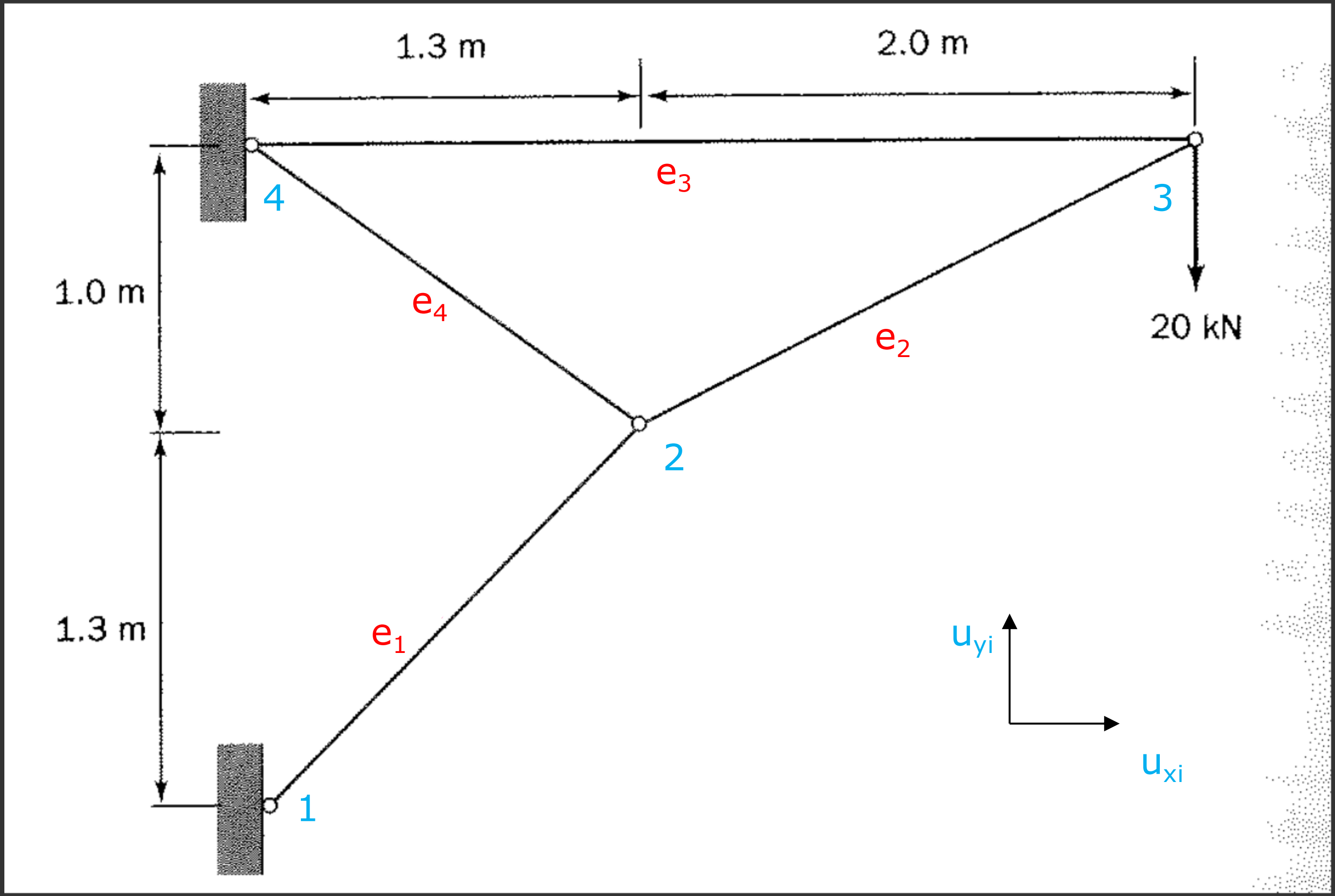


Figure Q1b

$$[K^e]_g = \left(\frac{AE}{L}\right)\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Truss Problem

- Construct the stiffness matrix of the structure shown below



Problem 17.4 from Benham, Crawford & Armstrong