The University of Nottingham

UNITED KINGDOM · CHINA · MALAYSIA

Mechanics of Solids

MMME2053

Thick Cylinders

Lecture 1

Learning Objectives

1. Appreciate the difference between the stress analysis of thin and thick cylinders (knowledge)
2. Understand the derivation of Lame’s equations (comprehension);
3. Determine the stresses in a thick walled cylinder subjected internal and external pressure (application);
4. Determine the stresses caused by shrink fitting a cylinder onto another (application);
5. Be able to include ‘inertia’ effects into the thick cylinder equations to calculate the stresses in a rotating disc (application).

Thick Cylinders

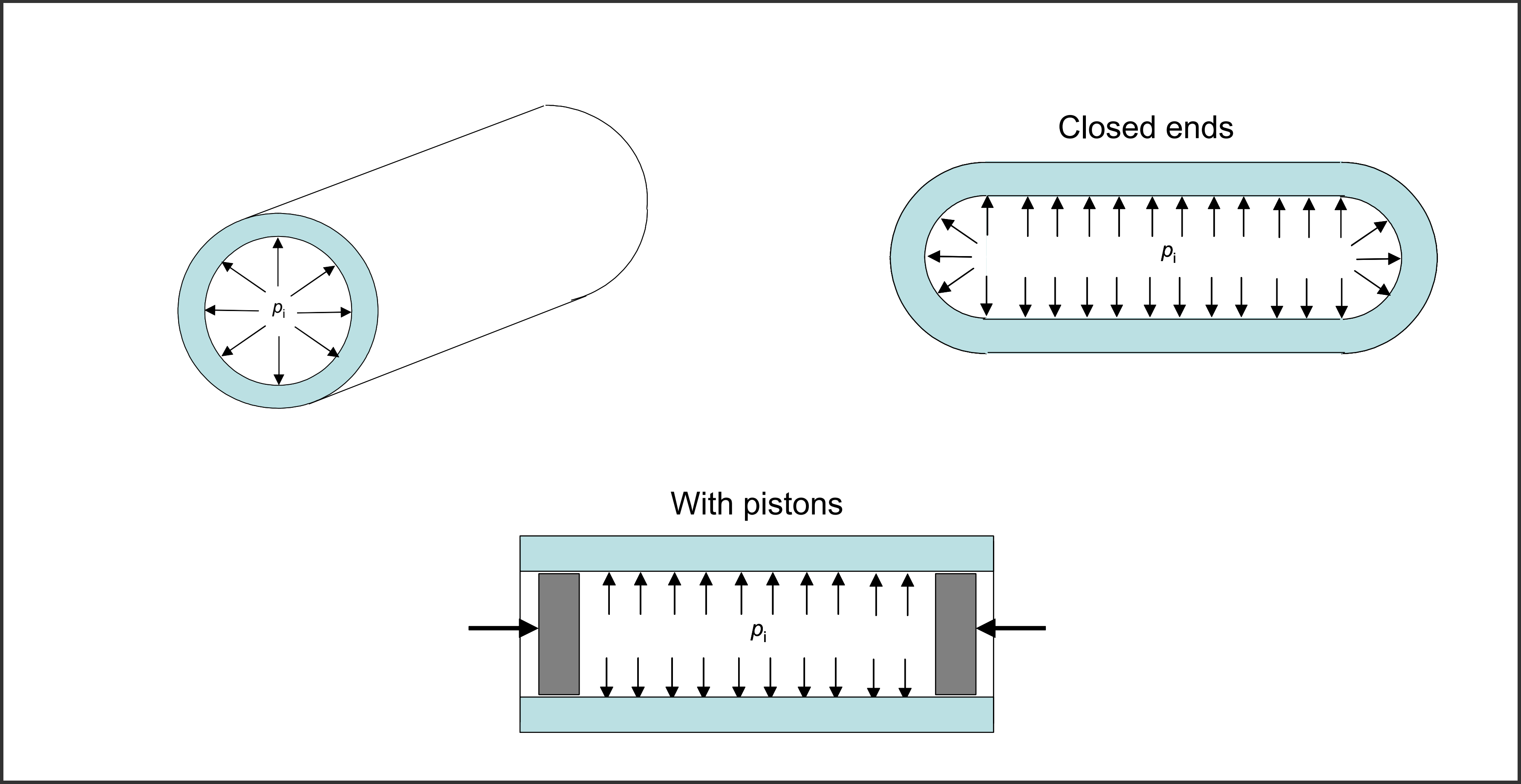
- This cylinders are widely used in the chemical, petroleum and military industries as well as power plants (both fossil and nuclear).

- They are usually subjected to high pressure & temperatures which may be constant or cycling.

- Industrial problems often witness ductile fracture of materials due to some discontinuity in geometry or material characteristics.

Thick Cylinders

- Examples:



Thick Cylinders

- Three cases that we need to consider in this section:

1. Thick walled cylinders subjected to internal and external pressure
2. Shrink fitting of cylinders
3. Stresses in a rotating disc

Thin Cylinders Recap

- For an internally pressurised thin cylinder:
  - the variations of the stresses through the wall thickness are negligible (assumption)
  - problem is *statically determinate*, i.e. expressions for the stresses can be obtained by consideration of equilibrium alone

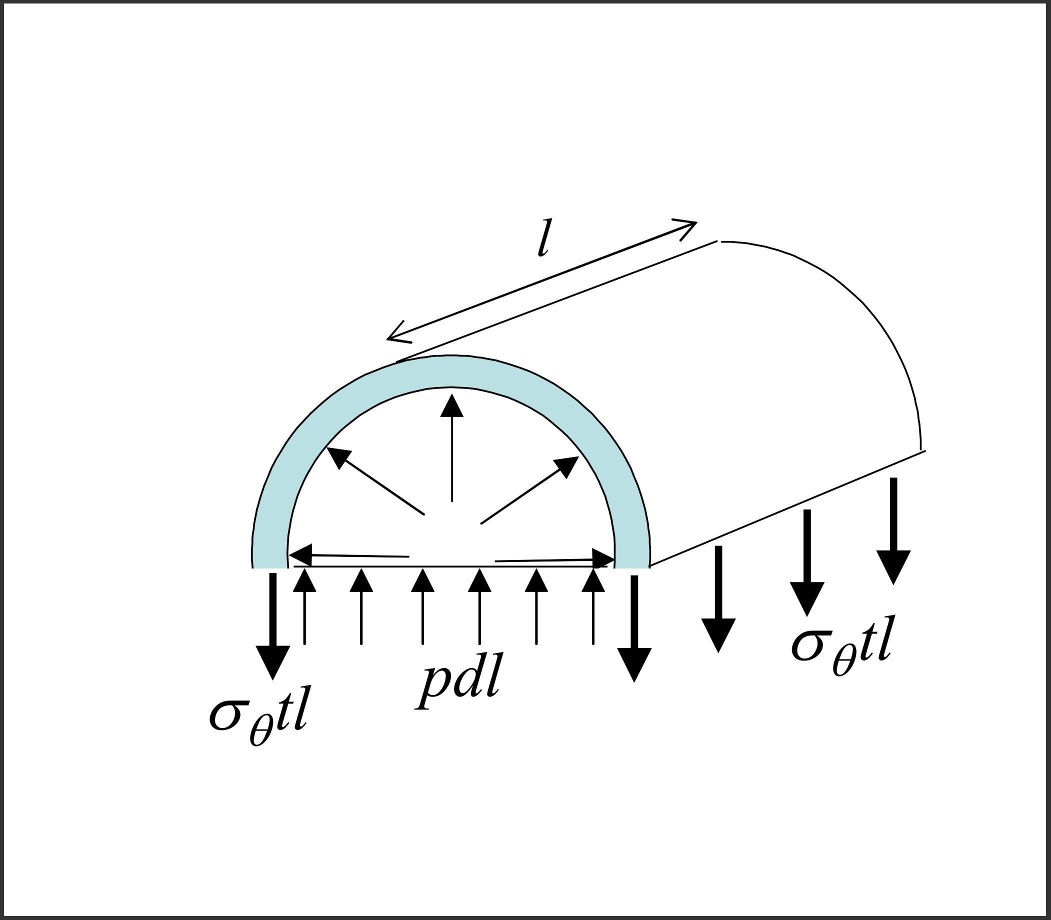
- Thin cylinders are normally those with $t/R < 1/10$

Thin Cylinders Recap

Hoop Stress

$$2\sigma_\theta tl = pdl$$

$$\sigma_\theta = \frac{pd}{2t} = \frac{pR}{t}$$

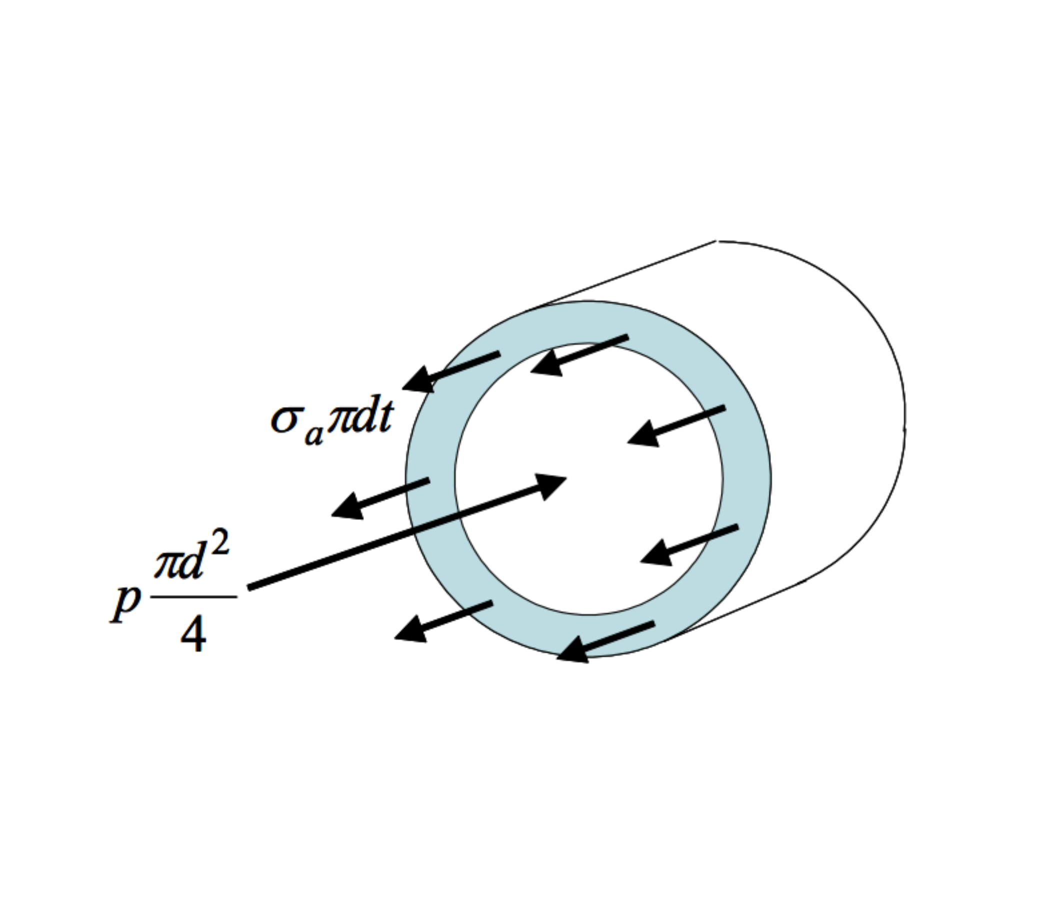


Thin Cylinders Recap

Axial Stress

$$\sigma_a \pi d t = p\frac{\pi d^2}{4}$$

$$\sigma_a = \frac{pd}{4t} = \frac{pR}{2t}$$



Thick Cylinders

- Thick cylinders differ from thin cylinders in that the variation of stress through the wall thickness is significant when subjected to internal and/or external pressure whereas for thin cylinders, the variation of stress is negligible.

Analysis of Thick Cylinders

- Thick cylinder problems are *statically indeterminate*
  - In order to obtain a solution it is necessary to consider:
    - Equilibrium,
    - Compatibility
    - Material behaviour (stress-strain relationship)

Assumptions

i. Plane transverse sections remain plane (this is true remote from the ends)
ii. Deformations are small
iii. The material is linear elastic, homogenous and isotropic.

Analysis of Thick Cylinders

- The hoop and radial stresses at any point (radius, *r*) in the wall cross-section of a thick cylinder can be determined using *Lame's equations*:

$$\sigma_h = A + \frac{B}{r^2}$$

$$\sigma_r = A - \frac{B}{r^2}$$

- Where *A* and *B* are *Lame's constants* (constants of integration)

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