The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Mechanics of Solids

MMME2053

Thick Cylinders

Lecture 3

Analysis of Thick Cylinders

- The hoop and radial stresses at any point (radius, *r*) in the wall cross-section of a thick cylinder can be determined using *Lame's equations*:

$$\sigma_r = A - \frac{B}{r^2}$$

$$\sigma_\theta = A + \frac{B}{r^2}$$

- Where *A* and *B* are *Lame's constants* (constants of integration)

- Note that, since $\varepsilon_z = \text{const}$ and $\sigma_r + \sigma_\theta = \text{const}$, then Eq. (9) shows that $\sigma_z = \text{const}$, i.e. it is independent of *r*. The value of σ_z can therefore be obtained from a consideration of axial equilibrium.

Analysis of Thick Cylinders

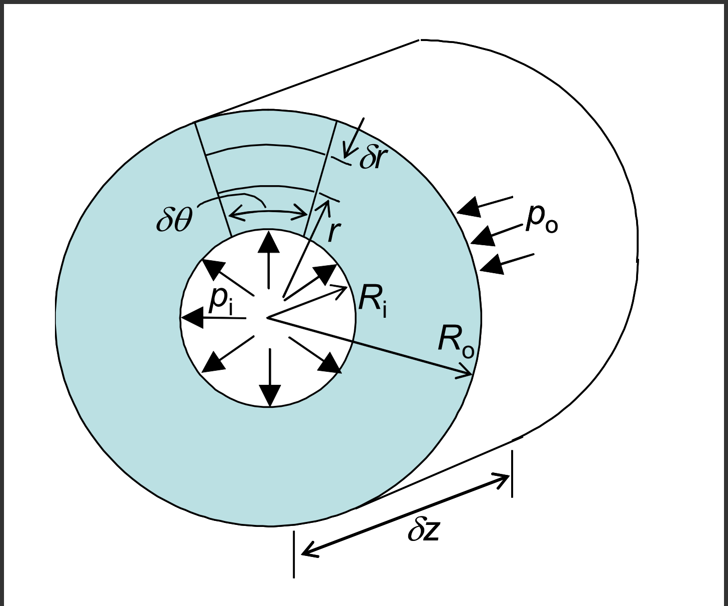
- We can obtain the values of Lame's constants from the boundary conditions of the problem

- At $r = R_i$

$$\sigma_r = -p_i$$

- At $r = R_o$

$$\sigma_r = -p_o$$



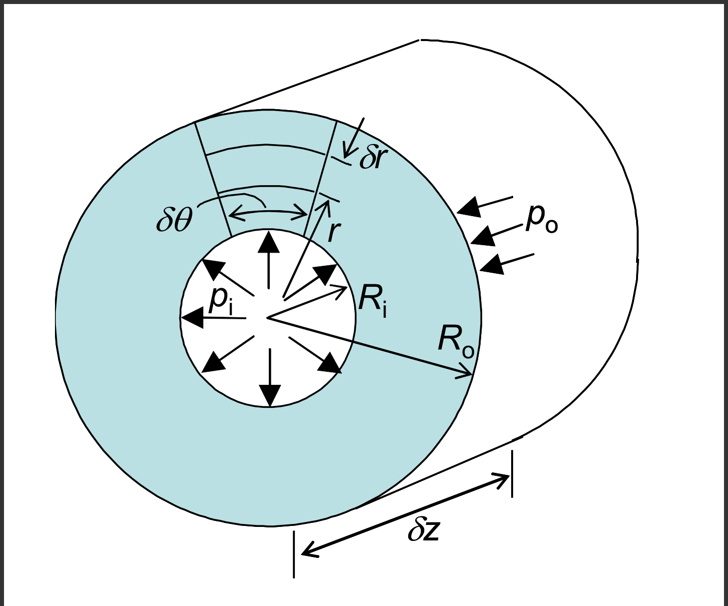
Analysis of Thick Cylinders

- We can obtain the values of Lame's constants from the boundary conditions of the problem

- Therefore:

$$-p_i = A - \frac{B}{R_i^2}$$

$$-p_o = A - \frac{B}{R_o^2}$$



- And *A* and *B* can be determined

Analysis of Thick Cylinders

- For a cylinder with closed ends, simple axial equilibrium leads us to:

$$\sigma_z \pi (R_o^2 - R_i^2) = \pi R_i^2 p_i - \pi R_o^2 p_o$$

$$\sigma_z = \frac{R_i^2 p_i - R_o^2 p_o}{(R_o^2 - R_i^2)}$$

- While for a cylinder with pistons, no axial load is transferred to the cylinder:

$$\sigma_z = 0$$

Analysis of Thick Cylinders

- For a solid cylinder, $R_i = 0$:

$$\sigma_r = A - \frac{B}{0^2} = \infty$$

- Unless $B = 0$. So in this case, as the stresses cannot be infinite, the radial and hoop stresses are equal and are constant

$$\sigma_r = \sigma_\theta = A$$

Learning Objectives

1. Appreciate the difference between the stress analysis of thin and thick cylinders (knowledge);
2. Understand the derivation of Lame’s equations (comprehension);
3. Determine the stresses in a thick walled cylinder subjected internal and external pressure (application);
4. Determine the stresses caused by shrink fitting a cylinder onto another (application);
5. Be able to include ‘inertia’ effects into the thick cylinder equations to calculate the stresses in a rotating disc (application).