Mechanics of Solids

Thermal Stress and Strain

Lecture 1

Learning Objectives

1. Recall that thermal strains arise when a change in temperature is applied to an unconstrained body (knowledge);

2. Recognise the cause of thermal strains and how ‘thermal stresses’ are caused by thermal strains (comprehension);

3. Solve problems involving both mechanical and thermal loading (application).

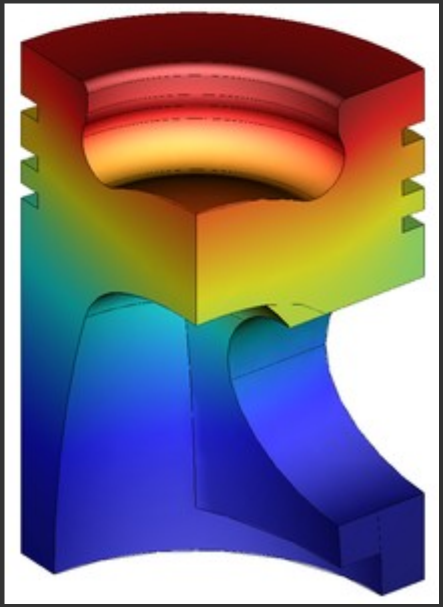
Introduction

Changes of temperature in a body cause expansion and contraction

Thermal stresses and strains are important in many engineering situations

For example:

Power plant, aeroengines, IC engines and many others



Diesel Engine Piston

Introduction

- Changes of temperature in a body cause expansion/contraction.
- This phenomenon is quantified by the coefficient of thermal expansion, α.
- Some typical values of thermal expansion coefficient for some common engineering materials are presented in Table 1.
- For isotropic materials, α is the same for all directions.

Introduction

Material	Coefficient of Thermal Expansion, α, [°C^{-1}]
Concrete	10×10^{-6}
Steel	11×10^{-6}
Aluminium	23×10^{-6}
Nylon	144×10^{-6}
Rubber	162×10^{-6}

Notice that the values of steel and concrete are very similar – this is good news for reinforced concrete used in buildings

Introduction

- For a bar of length l, subjected to a temperature change ΔT, the change in length $\delta l_{thermal}$ due to the temperature change is given by:

$$\delta l_{thermal} = l\alpha\Delta T$$

- The thermal strain due to this length change can be determined as follows:

$$\varepsilon_{thermal} = \frac{\delta l_{thermal}}{l} = \frac{l\alpha\Delta T}{l} = \alpha\Delta T$$

Introduction

- Using the principle of superposition, which states that:

$$\begin{bmatrix}The\ total\ effects\ of\ combined\\ loads\ applied\ to\ a\ body\end{bmatrix} = \sum \begin{bmatrix}The\ effects\ of\ the\ individual\\ loads\ applied\ separately\end{bmatrix}$$

thermal extensions can simply be added to elastic (mechanical) extensions to give the total extension by:

$$\delta l_{total} = \delta l_{elastic} + \delta l_{thermal}$$

For our uniaxial bar:

$$\delta l_{total} = \frac{FL}{AE} + l\alpha\Delta T$$

Introduction

http://www.youtube.com/watch?v=aT4tkbM38Fg

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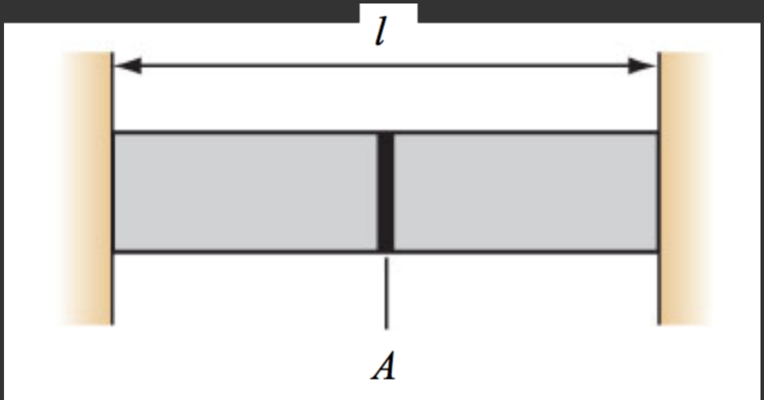
Resistive Heating of a Bar

- The bar shown below is subjected to a temperature rise of ΔT and restricted from expanding by constraints at each end.



- Since the bar cannot extend:

$$\delta l_{total} = \delta l_{elastic} + \delta l_{thermal} = 0$$

Resistive Heating of a Bar

- Or:

$$\delta l_{total} = \frac{FL}{AE} + l\alpha\Delta T = 0$$

- Cancelling through l and rearranging for the reaction force, F, gives:

$$F = -AE\alpha\Delta T$$

- And we can determine the stress using:

$$\sigma = \frac{F}{A} = -E\alpha\Delta T$$