Mechanics of Solids

Thermal Stress and Strain

Lecture 2

Learning Objectives

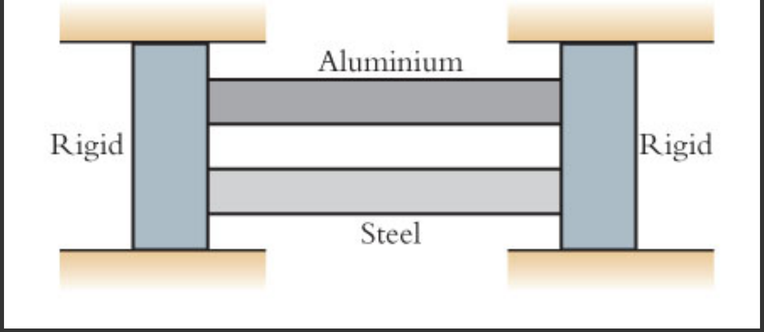
1. Recall that thermal strains arise when a change in temperature is applied to an unconstrained body (knowledge);

2. Recognise the cause of thermal strains and how ‘thermal stresses’ are caused by thermal strains (comprehension);

3. Solve problems involving both mechanical and thermal loading (application).

Compound Bar Assembly

The compound bar assembly



is subjected to a temperature change ΔT, will the bars be in tension or compression?

Compound Bar Assembly

- We can consider this intuitively
- The extension of the bars must be equal:

$$\delta l_{steel} = \delta l_{alu}$$

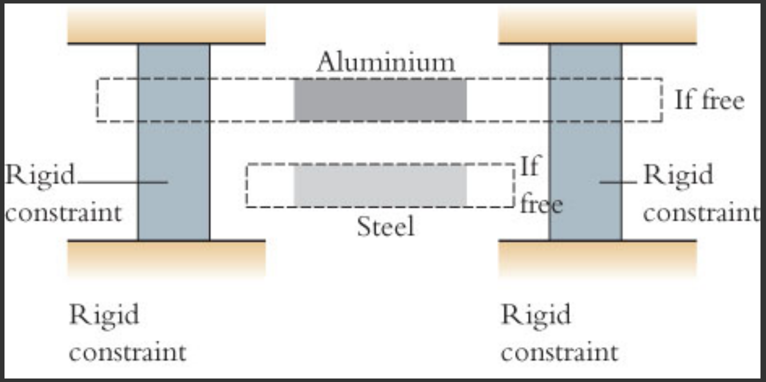
Material	Coefficient of Thermal Expansion, α, [°C^{-1}]
Concrete	10×10^{-6}
Steel	11×10^{-6}
Aluminium	23×10^{-6}
Nylon	144×10^{-6}
Rubber	162×10^{-6}

$$\alpha_{alu} > \alpha_{steel}$$

Compound Bar Assembly

- The aluminium bar will want to extend more than the steel bar but is constrained from doing so due to the rigid end blocks attached to the steel bar. This means that the **aluminium bar will be in compression**.

- The reverse is true of the steel bar, it wants to extend less than the aluminium bar but the rigid end blocks attached the aluminium bar forces it to extend further, therefore the **steel bar is in tension**.



Compound Bar Assembly

- Considering the problem analytically, the change in lengths are given by:

$$\frac{F_{steel}l}{A_{steel}E_{steel}} + l\alpha_{steel}\Delta T = \frac{F_{alu}l}{A_{alu}E_{alu}} + l\alpha_{alu}\Delta T$$

- Considering equilibrium using the FBD:

$$F_{steel} = -F_{alu}$$

Compound Bar Assembly

- Substituting in for F_{steel} and rearranging:

$$l\Delta T(\alpha_{steel} - \alpha_{alu}) = F_{alu} l \left[\frac{1}{A_{alu}E_{alu}} + \frac{1}{A_{steel}E_{steel}}\right]$$

- Therefore:

$$\sigma_{alu} = \frac{F_{alu}}{A_{alu}} = \frac{\Delta T(\alpha_{steel} - \alpha_{alu})}{\left[\frac{1}{E_{alu}} + \frac{A_{alu}}{A_{steel}E_{steel}}\right]}$$

- As $\alpha_{steel} < \alpha_{alu}$ this means that $\sigma_{alu} < 0$ i.e. the **aluminium bar is in compression**.

Compound Bar Assembly

- From the force equilibrium:

$$A_{alu}\sigma_{alu} = -A_{steel}\sigma_{steel}$$

- Then:

$$\sigma_{steel} = -\frac{A_{alu}\sigma_{alu}}{A_{steel}} = -\frac{A_{alu}}{A_{steel}}\frac{\Delta T(\alpha_{steel} - \alpha_{alu})}{\left[\frac{1}{E_{alu}} + \frac{A_{steel}}{A_{steel}E_{steel}}\right]}$$

- As $\alpha_{steel} < \alpha_{alu}$ this means that $\sigma_{steel} > 0$ i.e. the **steel bar is in tension**.

Generalised Hooke's Law including Thermal Strains

$$\varepsilon_{total} = \varepsilon_{mech} + \varepsilon_{thermal}$$

$$\varepsilon_{mech,x} = \frac{1}{E}\left(\sigma_x - \nu\left[\sigma_y + \sigma_z\right]\right)$$

$$\varepsilon_{thermal} = \alpha \Delta T$$

Generalised Hooke's Law including Thermal Strains

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left[\sigma_y + \sigma_z\right]\right) + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left[\sigma_x + \sigma_z\right]\right) + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left[\sigma_x + \sigma_y\right]\right) + \alpha\Delta T$$

$$\gamma_{xy} = \tau_{xy}/G \quad \gamma_{yz} = \tau_{yz}/G \quad \gamma_{zx} = \tau_{zx}/G$$

This allows us to tackle more complex thermal stress/strain problems