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Mechanics of Solids

Thermal Stress and Strain

Lecture 3

Learning Objectives

1. Recall that thermal strains arise when a change in temperature is applied to an unconstrained body (knowledge);

2. Recognise the cause of thermal strains and how ‘thermal stresses’ are caused by thermal strains (comprehension);

3. Solve problems involving both mechanical and thermal loading (application).

Generalised Hooke's Law including Thermal Strains

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left[\sigma_y + \sigma_z\right]\right) + \alpha\Delta T$$

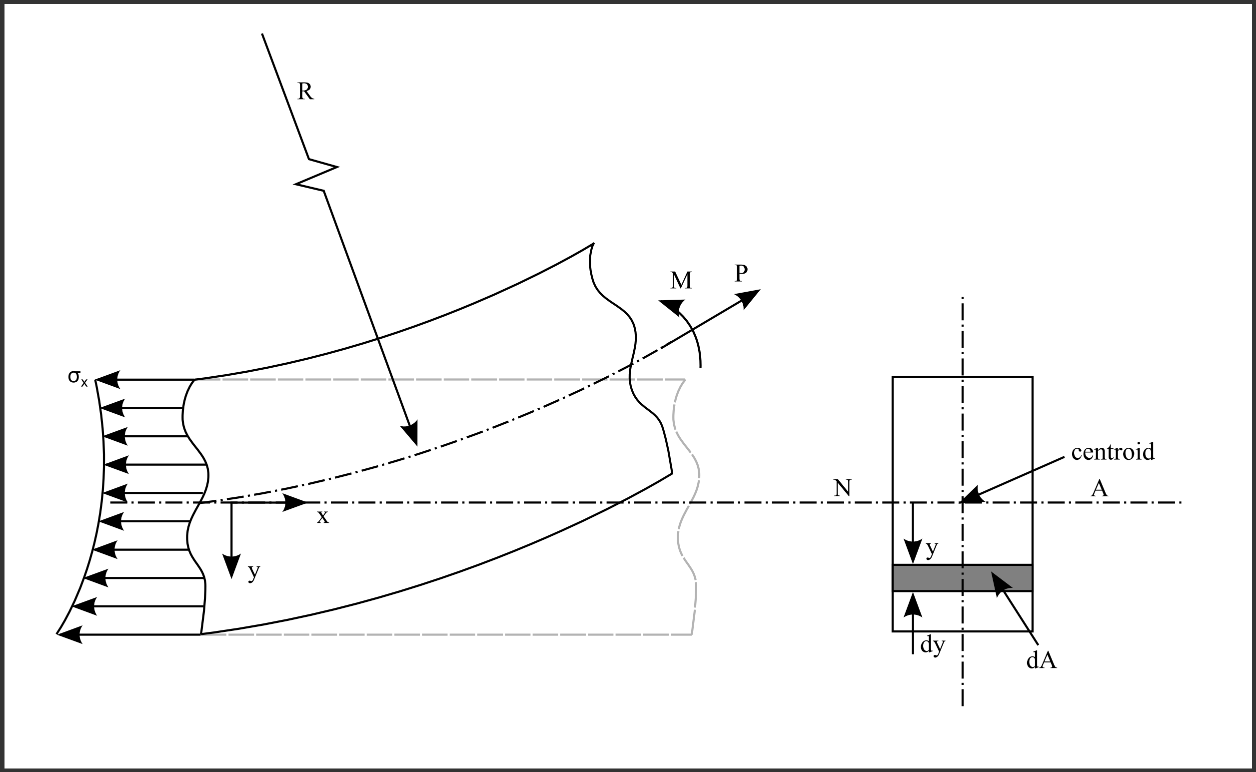
$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu[\sigma_x + \sigma_z]\right) + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left[\sigma_x + \sigma_y\right]\right) + \alpha\Delta T$$

$$\gamma_{xy} = \tau_{xy}/G \quad \gamma_{yz} = \tau_{yz}/G \quad \gamma_{zx} = \tau_{zx}/G$$

This allows us to tackle more complex thermal stress/strain problems

Case 1: An initially straight uniform beam



Determine deformations and stresses (small deformations)

The temperature change is (assumed) purely a function of y, i.e. $\Delta T = \Delta T(y)$.

The coefficient of thermal expansion, α. Axial force *P*, and pure bending, about the z-z axis, *M*, are also applied.

$\sigma_z, \sigma_y, \tau_{xz}$ and $\tau_{yz} = 0$ because the cross-sectional dimensions are small compared with the length.

Also,

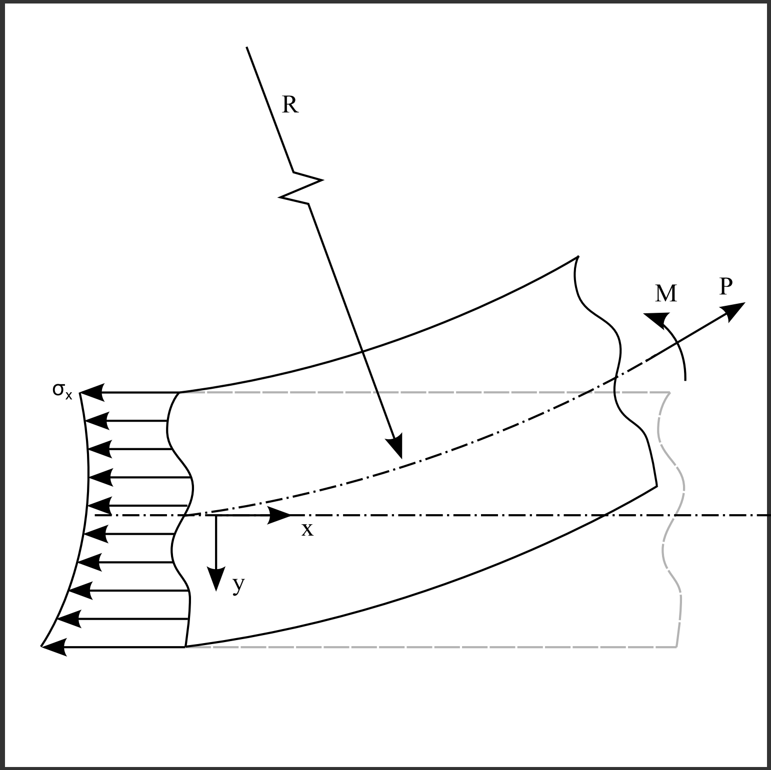
$\tau_{xy} = 0$ because *M* does not vary with x, $S = \frac{dM}{dx} = 0$

Compatibility

Remote from the ends, strain varies linearly with *y*,

$$\varepsilon_x = \bar{\varepsilon} + \frac{y}{R} \quad (1)$$

Where $\bar{\varepsilon}$ is the mean strain (at *y* = 0) and *R* is the radius of curvature.



Stress-strain

From the generalised Hooke's Law equation $\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left[\sigma_y + \sigma_z\right]\right) + \alpha\Delta T$

$$\varepsilon_x = \frac{\sigma_x}{E} + \alpha\Delta T$$ (2) – (as σ_y and σ_z are 0)

Substituting (1) into (2) and rearranging gives:

$$\sigma_x = E\left(\bar{\varepsilon} + \frac{y}{R} - \alpha\Delta T\right)$$ (3)

Axial Force Equilibrium

$$P = \int_A \sigma_x dA \quad (4)$$

Substituting (3) into (4) gives:

$$P = E\int_A \left(\bar{\varepsilon} + \frac{y}{R} - \alpha\Delta T\right) dA$$

$$P = E\bar{\varepsilon}A + \frac{E}{R}\int_A y dA - E\alpha\int_A \Delta T dA$$

As the axis passes through the centroid, $\int_A y dA = 0$, leaving:

$$P = E\bar{\varepsilon}A - E\alpha\int_A \Delta T dA \quad (5)$$

Moment Equilibrium

$$M = \int_A y\sigma_x dA \quad (6)$$

Substituting (3) into (6) gives:

$$M = E\int_A \left(\bar{\varepsilon} + \frac{y}{R} - \alpha\Delta T\right) ydA$$

$$M = E\bar{\varepsilon}\int_A ydA + \frac{E}{R}\int_A y^2 dA - E\alpha\int_A \Delta T ydA$$

By definition, $\int_A y^2 dA = I$, therefore: ($\int_A ydA = 0$ as before)

$$M = \frac{EI}{R} - E\alpha\int_A \Delta T ydA \quad (7)$$