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Mechanics of Solids

Thermal Stress and Strain

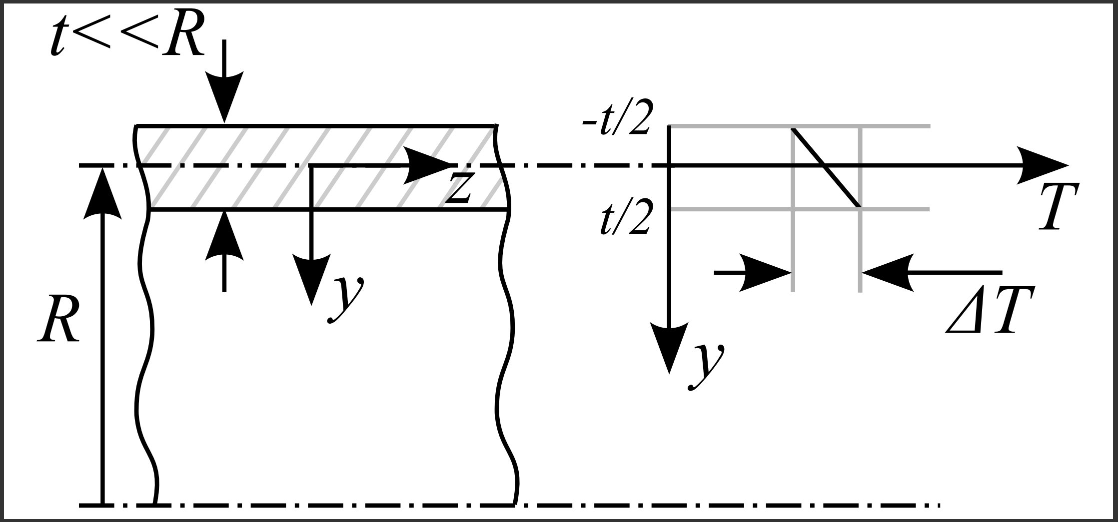
Lecture 4

Case 2: Thin Cylinders

Thin cylinders are in common use in power and chemical plant, e.g. pipes, pressure vessels, etc.

Often temperature variations are approximately linear through the thickness.

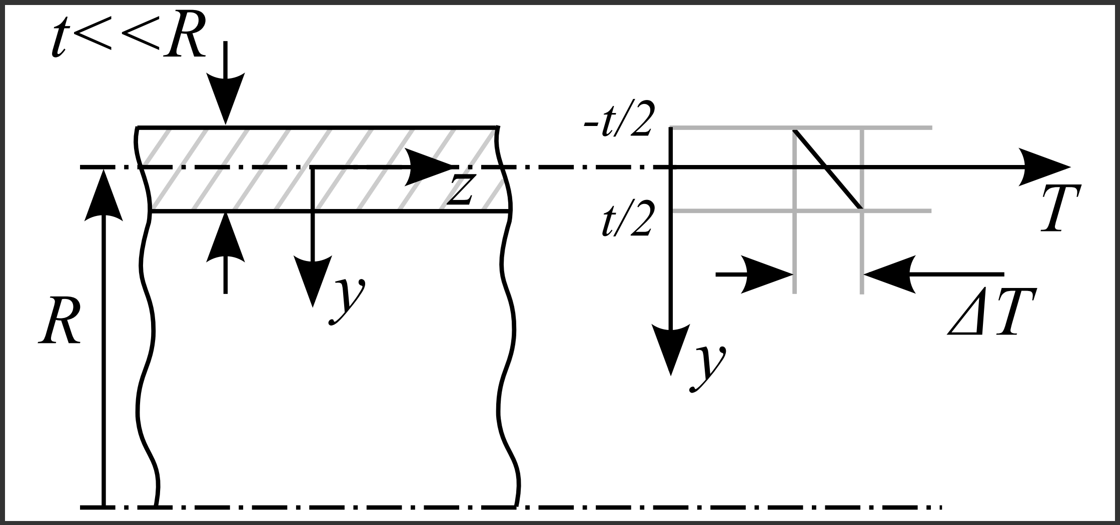
Considering positions remote from ends, flanges, etc.



It is convenient to consider the effect of the uniform temperature change and the temperature gradient separately.

If the cylinder is not restrained, then the uniform temperature change causes overall dimensional changes, but no stress.

The stresses due to axial restraint are easily calculated.



The temperature gradient is given by:

$$\Delta T(y) = \Delta T_{wall} \frac{y}{t}$$

and for a thin cylinder

$$\sigma_r \approx 0$$

Now using a cylindrical coordinate system:

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_z) + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_\theta) + \alpha\Delta T$$

As $\sigma_r \approx 0$

Away from the end of the cylinder, sections remain plane and circular.

From compatibility considerations (with no mean temperature change), the hoop and axial strains must both be zero.

Which gives:

$$\varepsilon_\theta = 0 = \frac{1}{E}(\sigma_\theta - \nu\sigma_z) + \alpha\Delta T_{wall}\frac{y}{t}$$

$$\varepsilon_z = 0 = \frac{1}{E}(\sigma_z - \nu\sigma_\theta) + \alpha\Delta T_{wall}\frac{y}{t}$$

Solving gives:

$$\sigma_\theta = \sigma_z = \frac{-E\alpha\Delta T_{wall}}{(1-\nu)}\frac{y}{t}$$

At y=t/2

$$\sigma_\theta = \sigma_z = \frac{-E\alpha\Delta T_{wall}}{2(1-\nu)}$$

At y=-t/2

$$\sigma_\theta = \sigma_z = \frac{E\alpha\Delta T_{wall}}{2(1-\nu)}$$