Mechanics of Solids

MMME2053

Shear Stresses

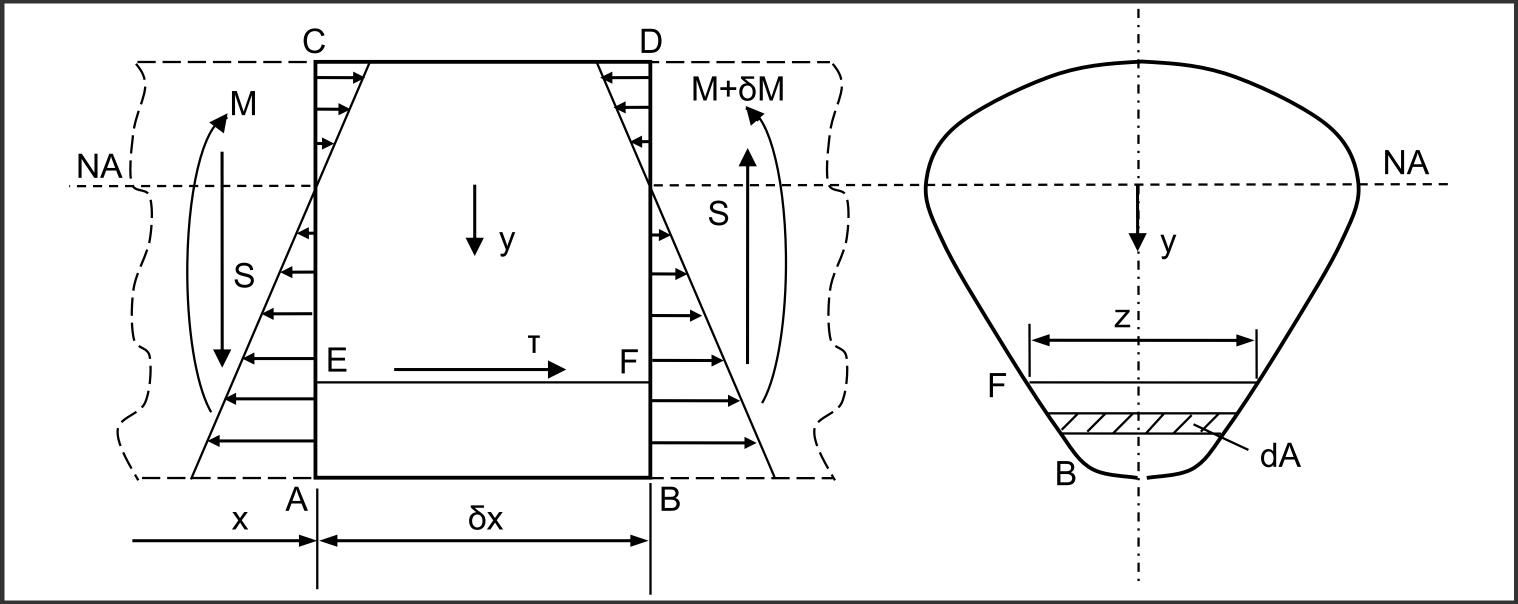
Lecture 2

Shear Stresses in Beams (1)

- The through-thickness shear force in a beam is the integral of the shear stresses over the cross-section
- We will derive an expression for the transverse (through thickness) shear stress at any position of a section in an arbitrary beam as a function of the shear force

Shear Stresses in Beams (2)

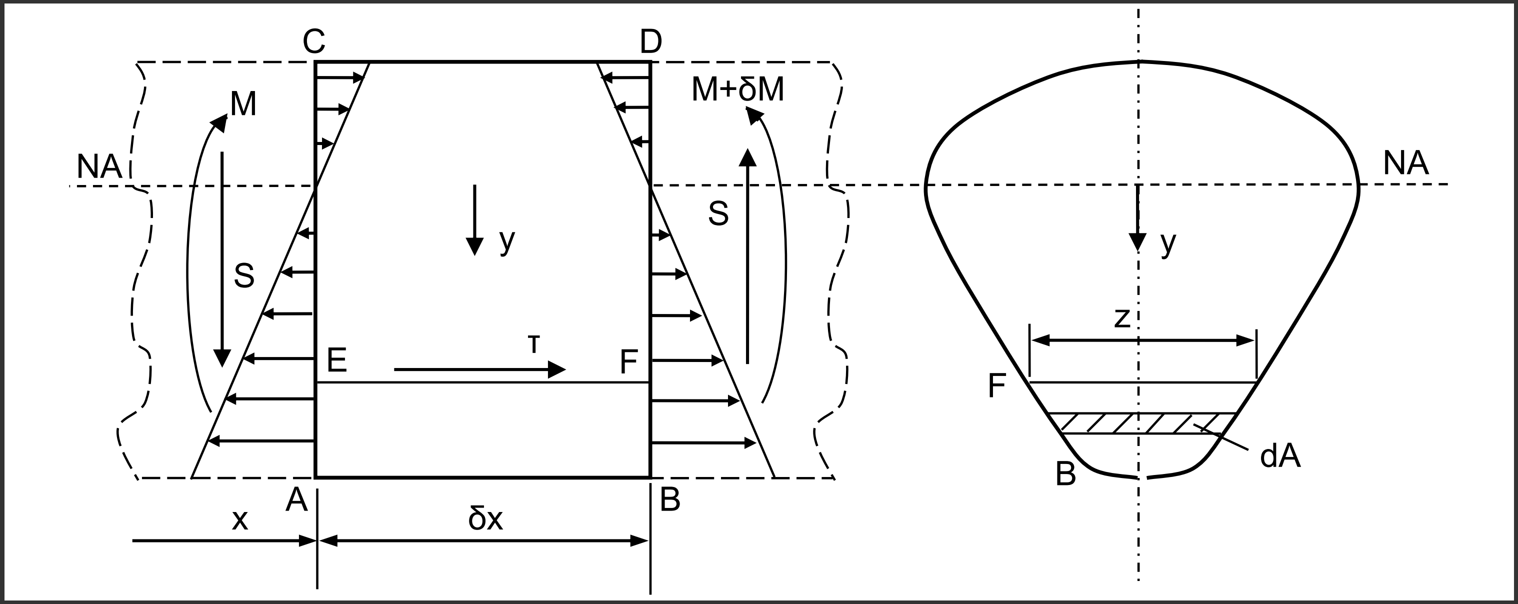
- Consider an element of beam length, δx, as shown below. The bending moment at x, section AC, is M and at $x + \delta x$, section BD, is $M + \delta M$.



- The direct bending stresses on AC are: $$\sigma_{AC} = \frac{My}{I}$$

Shear Stresses in Beams (3)

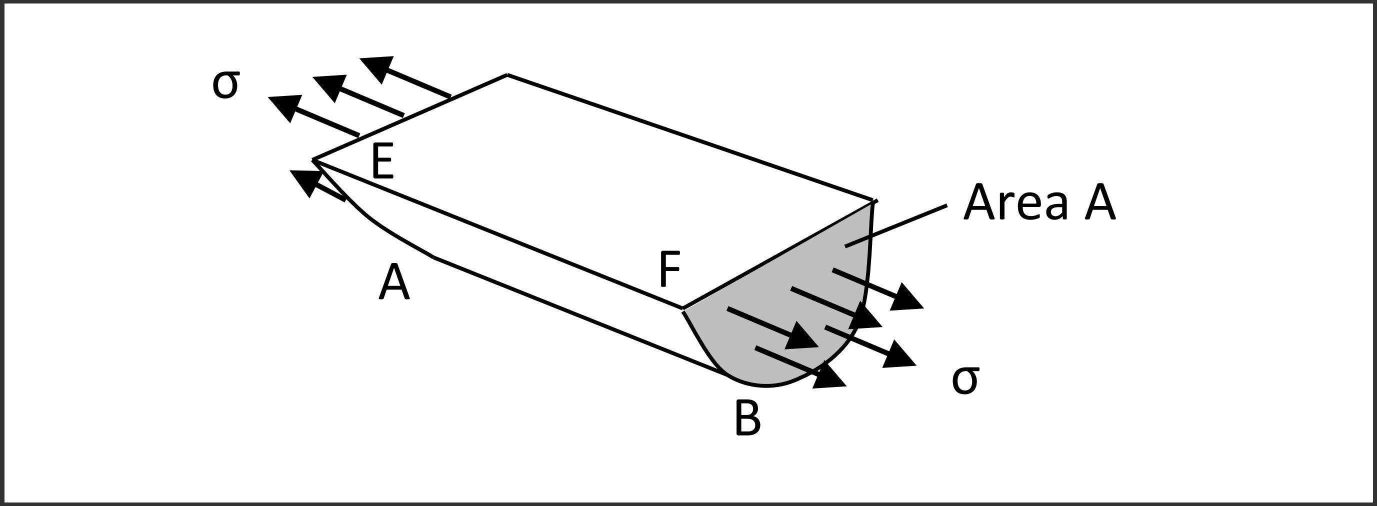
- And the direct bending stresses on BD are given by



$$\sigma_{BD} = \frac{(M + \delta M)y}{I}$$

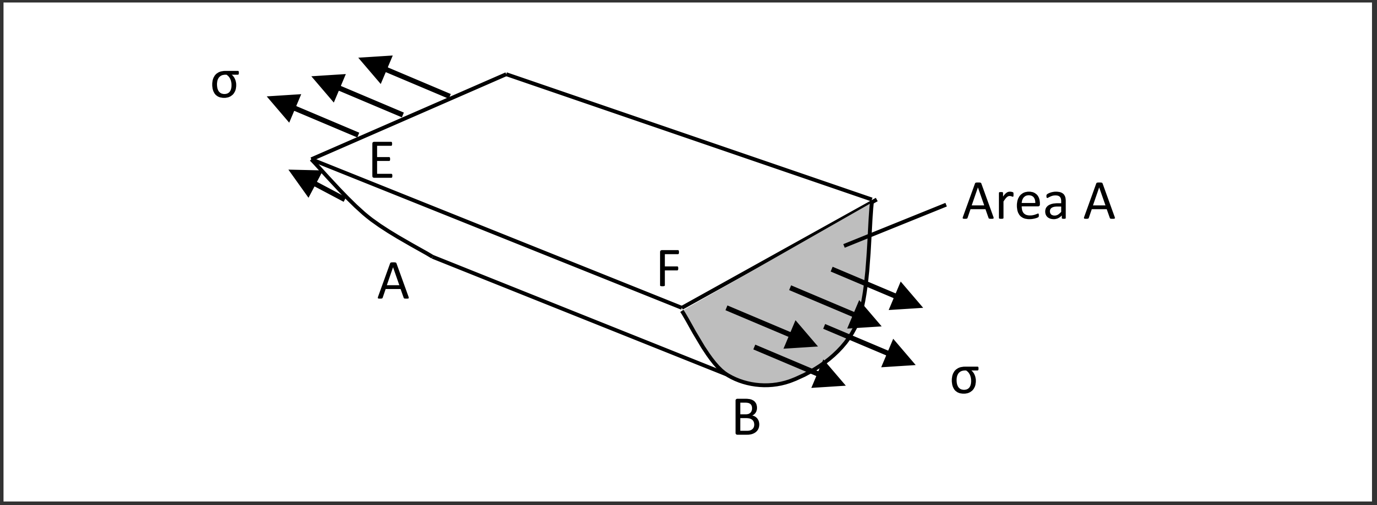
Shear Stresses in Beams (4)

- Thus, when the bending moment varies along the length of the beam on an element such as ABEF, there is a net axial force due to change in the bending stresses



Shear Stresses in Beams (5)

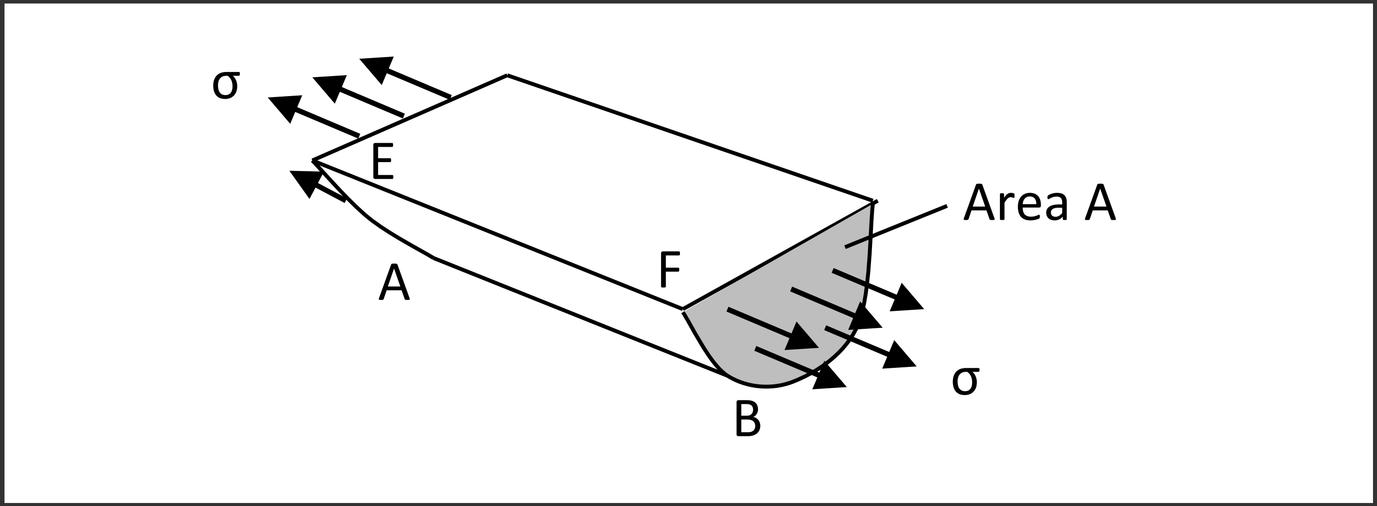
- The force on the face EA is the integral of the bending stresses over the area, *A*



$$F_{EA} = \int_A \frac{M}{I} y dA$$

Shear Stresses in Beams (6)

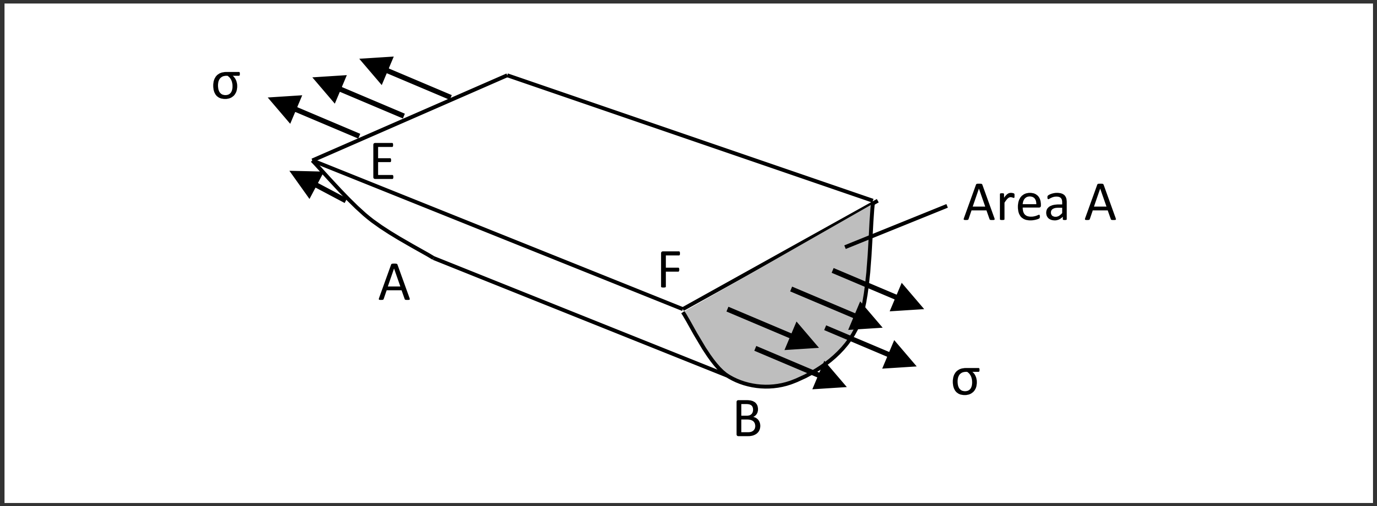
- The force on the face FB is the integral of the bending stresses over the area, *A*



$$F_{FB} = \int_A \frac{(M + \delta M)}{I} y dA$$

Shear Stresses in Beams (7)

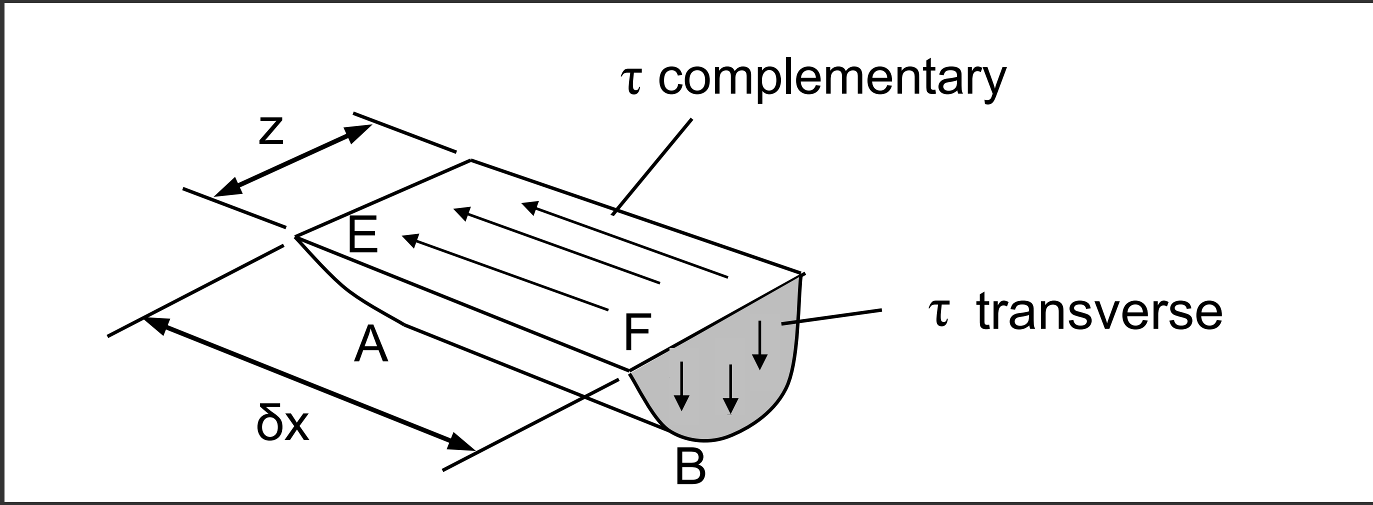
- The net force to the right acting on the element ABEF is the difference in these



$$\text{Net Force} = \int_A \frac{\delta M}{I} y dA \qquad [1]$$

Shear Stresses in Beams (8)

- In order to maintain equilibrium of ABEF, shear stresses must act on the plane EF, of average value τ. These shear stresses are complementary to the transverse shear stresses



- The net force to the left due to these complementary shear stresses is:

$$\text{Net Force} = \tau z \delta x$$

Shear Stresses in Beams (9)

- Equilibrium of ABEF requires the net force due to bending to balance the net force due to the complementary shear

$$\tau z \delta x + \int_A \frac{\delta M}{I} y dA = 0$$

$$\tau z \delta x = \frac{1}{Iz} \frac{\delta M}{\delta x} \int_A y dA$$

Shear Stresses in Beams (10)

- In the limit,

$$\frac{\lim}{\delta x \to 0}\frac{\delta M}{\delta x} = \frac{dM}{dx} = -S$$

- Where S is the shear force at the section

Shear Stresses in Beams (11)

- Which gives:

$$\tau = \frac{S}{Iz}\int_A ydA \qquad [2]$$

- This is the general integral expression for transverse shear stress at any position y through the thickness.

Shear Stresses in Beams (12)

- Which can be written in discrete form as:

$$\tau = \frac{SA\bar{y}}{Iz} \qquad [3]$$

- where A is the area of the part of the cross-section outside the position at which τ is determined, and $\bar{y}$ is the distance of the centroid of this area from the neutral axis.

Shear Stresses in Beams (13)

- May see it in some texts as:

$$\tau = \frac{SQ}{Iz}$$

- Where Q represents $A\bar{y}$ but is generally more applicable for complex sections with changes is cross-sectional area through the depth of the beam. We can calculate Q for each sub-area of the section and sum them together.

Shear Stresses in Beams (14)

- For a general beam cross section:

$$\tau = \frac{SA\bar{y}}{Iz}$$