Mechanics of Solids

MMME2053

Shear Stresses

Lecture 4

Learning Objectives

1. Appreciate that in addition to longitudinal bending stresses, beams also carry transverse shear stresses arising from the vertical shear loads acting within the beam (knowledge)
2. Be able to derive a general formula, in both integral and discrete form, for evaluating the shear stress distribution through a cross-section (comprehension);
3. Determine the shear stress distribution through the thickness in a rectangular, circular and I-section beam (application);

Learning Objectives

4. Understand that in an I-section, in addition to the transverse vertical shear stresses in the flange and web, more dominant horizontal shear stresses also occur in the flange (comprehension);
5. Recognise that the resultant of the shear stresses always act through one point, known as the 'shear centre' (comprehension);
6. Calculate the position of the shear centre (application);
7. Understand that if the applied loads do not act through the shear centre, then there is a resultant torsional load, which can result in twisting of the section if the torsional rigidity of the section is low e.g. thin walled sections (comprehension).

Important Points

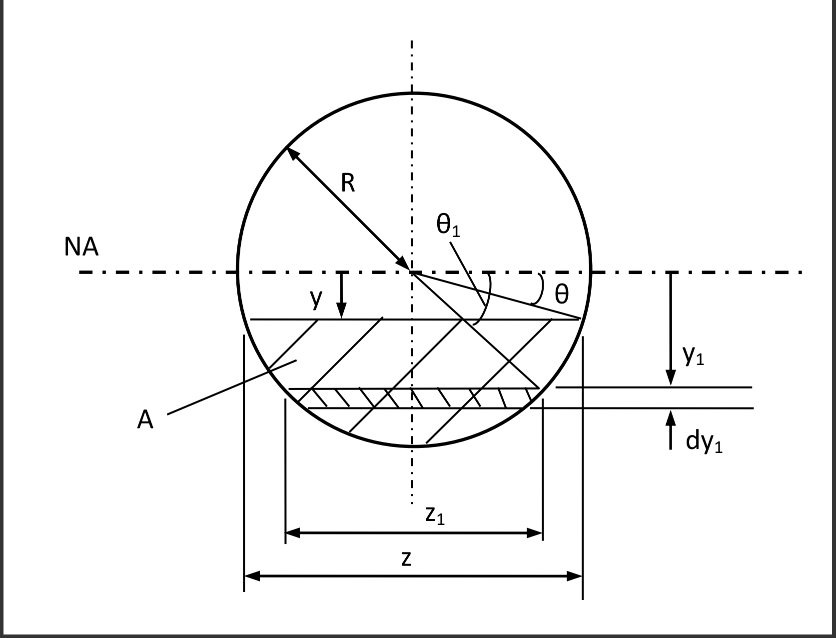
- The general formulae for shear stresses in beams in both integral and discrete forms are:

$$\tau = \frac{S}{Iz}\int_A ydA \qquad\qquad \tau = \frac{SA\bar{y}}{Iz}$$

Where S is the shear force on the section, I is the second moment of area, y is the position from the N.A. at which you wish_to determine the shear stress, z is the thickness of the section at that location, A is the area outside that location, and y is the distance to the centroid of that area.

Shear Stress Distribution in a Circular Beam (1)

- To calculate the transverse shear stress distribution in a circular cross section:



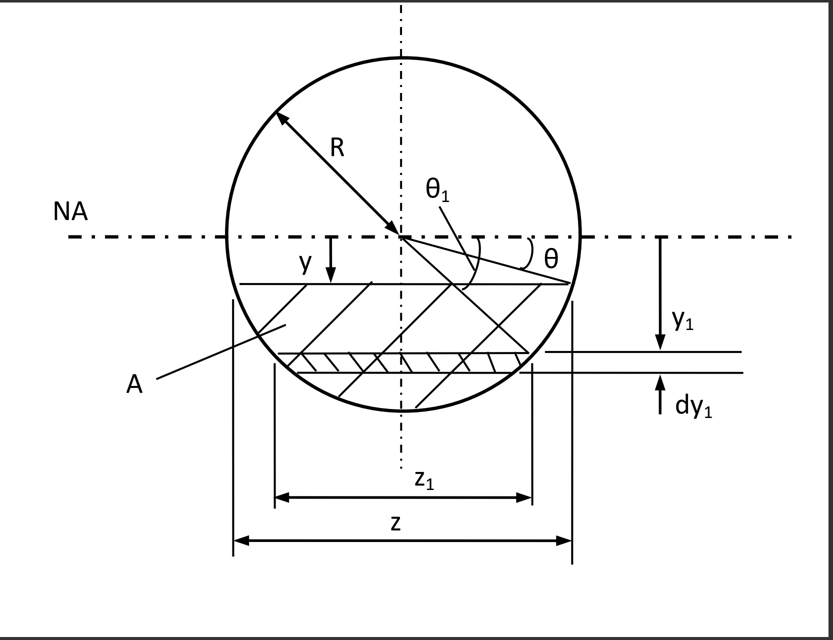
- We use the integral form of the shear equation ([2] in the handout)

Shear Stress Distribution in a Circular Beam (2)

- Because of the circular shape, it is convenient to change the variables *y* and *z* in this equation to polar variables, *R* and *θ*.

- So,

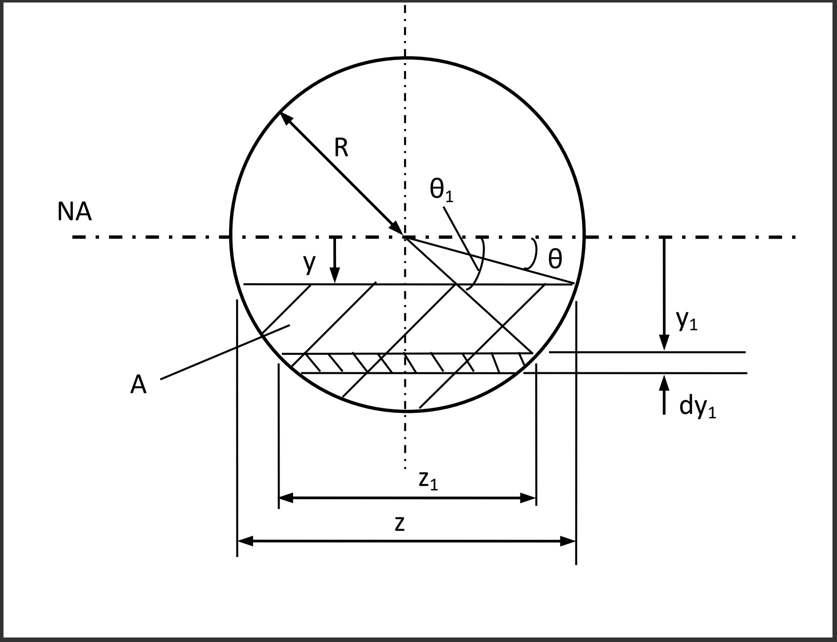
$$y_1 = R \sin\theta_1$$
$$dy_1 = R \cos\theta_1 d\theta_1$$
$$z_1 = 2R \cos\theta_1$$
$$z = 2R \cos\theta$$



Shear Stress Distribution in a Circular Beam (3)

- The second moment of area, I

$$I = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$$



Shear Stress Distribution in a Circular Beam (4)

- Substituting into the shear stress equation gives:

$$\tau = \frac{S}{Iz} = \int_A ydA = \frac{4S}{\pi R^4 2R\cos\theta}\int_0^{\frac{\pi}{2}} y_1 z_1 dy$$

$$= \frac{4S}{\pi R^4 2R\cos\theta}\int_0^{\frac{\pi}{2}} R_1 \sin\theta_1 \, 2R\cos\theta_1 \, R\cos\theta_1 d\theta$$

$$= \frac{4SR^3}{\pi R^5\cos\theta}\int_0^{\frac{\pi}{2}} \cos^2\theta_1 \sin\theta_1 d\theta$$

Shear Stress Distribution in a Circular Beam (5)

$$= \frac{4S}{\pi R^2 \cos\theta} \left[\frac{-\cos^3\theta}{3} \right]_0^{\frac{\pi}{2}}$$

$$\left(\cos\frac{\pi}{2} = 0 \right)$$

$$\tau = \frac{4S}{3\pi R^2} \cos^2\theta$$

Shear Stress Distribution in a Circular Beam (6)

- As:

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{y}{R}\right)^2$$

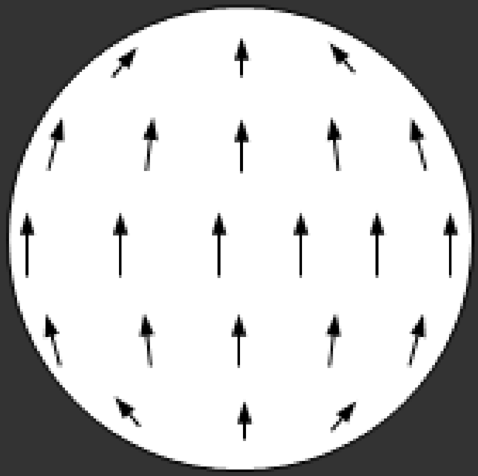
$$\tau = \frac{4S}{3\pi R^2}\left[1 - \left(\frac{y}{R}\right)^2\right]$$

Shear Stress Distribution in a Circular Beam (7)

- Again a parabolic distribution
- With a maximum value of τ at the N.A.(y = 0)

$$\tau_{max} = \frac{4S}{3\pi R^2} = \frac{4}{3}\tau_{av}$$

Shear Stress Distribution in a Circular Beam (8)



- In this case, τ must vary across the width of the section.
- At the free surface the shear stress must be zero. Therefore, the complementary shear on the cross-section, <u>normal to the boundary</u>, is also zero. Thus, shear must be tangential to the boundary as drawn.