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Mechanics of Solids

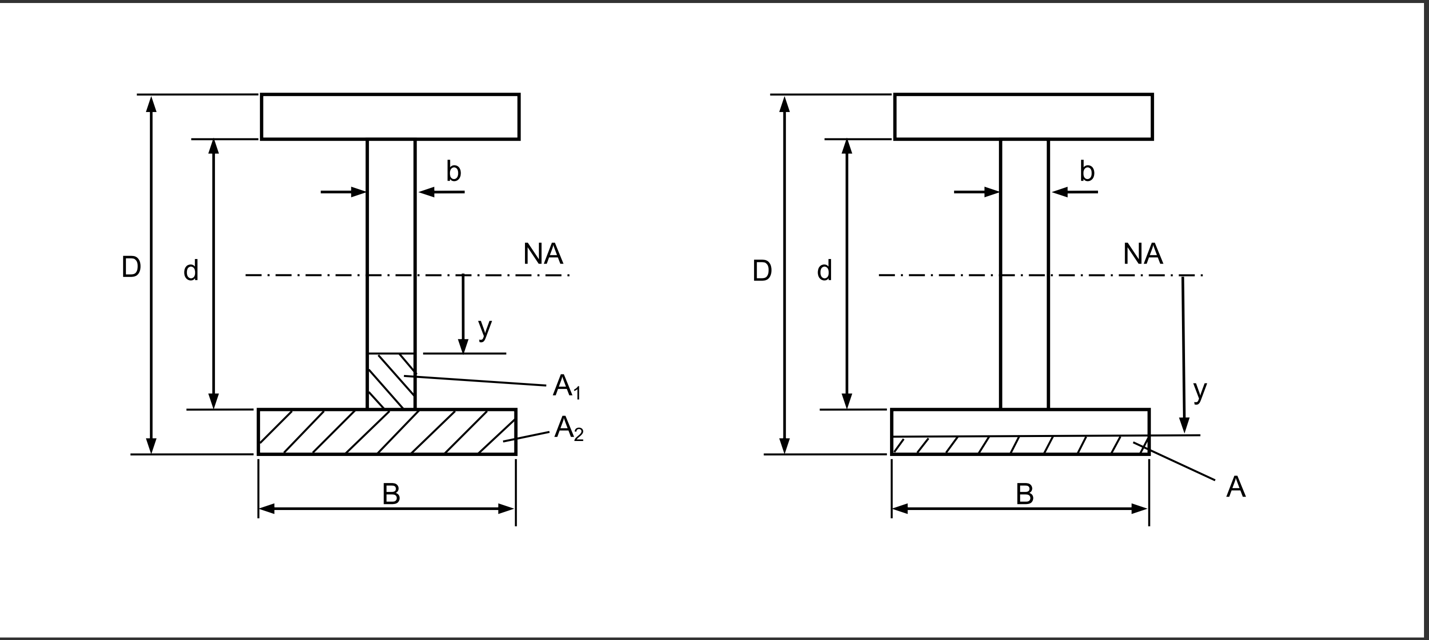
MMME2053

Shear Stresses

Lecture 5

Shear Stress Distribution in an I-Beam (1)

- Transverse shear stress down the centre line
- Need to consider web and flange sections separately using the discrete formula



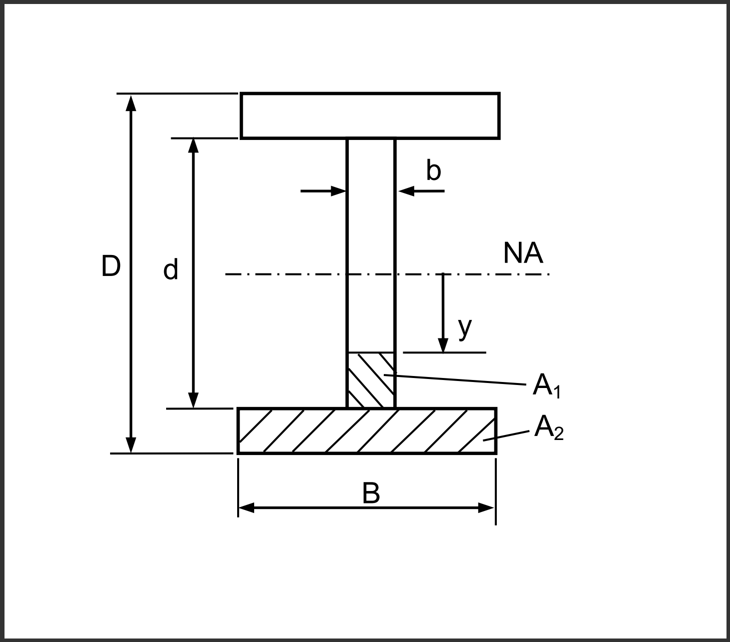
Shear Stress Distribution in an I-Beam (2)

- In the web:

$$\tau = \frac{SA\bar{y}}{Iz}$$

- However we have two discrete areas to consider:

$$\tau = \frac{SA\bar{y}}{Iz} = \frac{S}{Iz}\left[A_1\bar{y}_1 + A_2\bar{y}_2\right]$$



Shear Stress Distribution in an I-Beam (3)

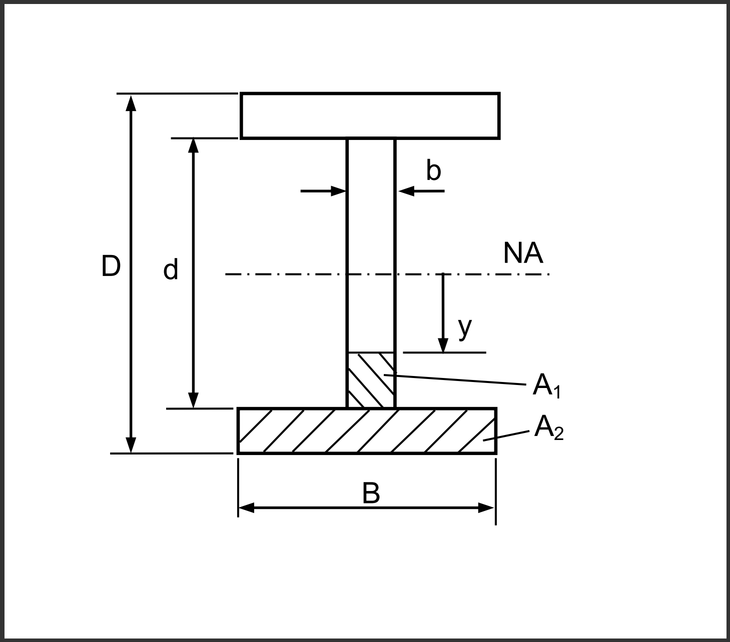
- So:

$$\tau = \frac{S}{Ib}\left[\begin{array}{l} \left(\frac{d}{2} - y\right) b \frac{1}{2}\left(\frac{d}{2} + y\right) \\ +B\left(\frac{D}{2} - \frac{d}{2}\right)\frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right) \end{array}\right]$$

$$\tau = \frac{S}{Ib}\left[\frac{b}{2}\left(\frac{d^2}{4} - y^2\right) + \frac{B}{2}\left(\frac{D^2}{4} - \frac{d^2}{4}\right)\right]$$

- And:

$$I = \frac{BD^3}{12} - \frac{(B-b)d^3}{12}$$



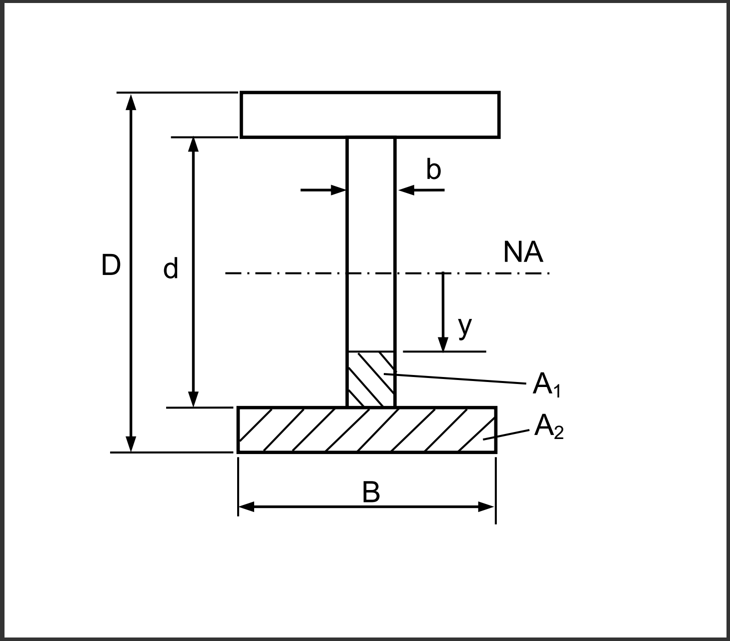
Shear Stress Distribution in an I-Beam (4)

- The maximum τ at y = 0

$$\tau_{max} = \frac{S}{Ib}\left[\frac{BD^2}{8} - \frac{(B-b)d^2}{8}\right]$$

- And at the bottom and top of the web y = +/- $d/2$

$$\tau = \frac{S}{Ib}\frac{B}{8}(D^2 - d^2)$$



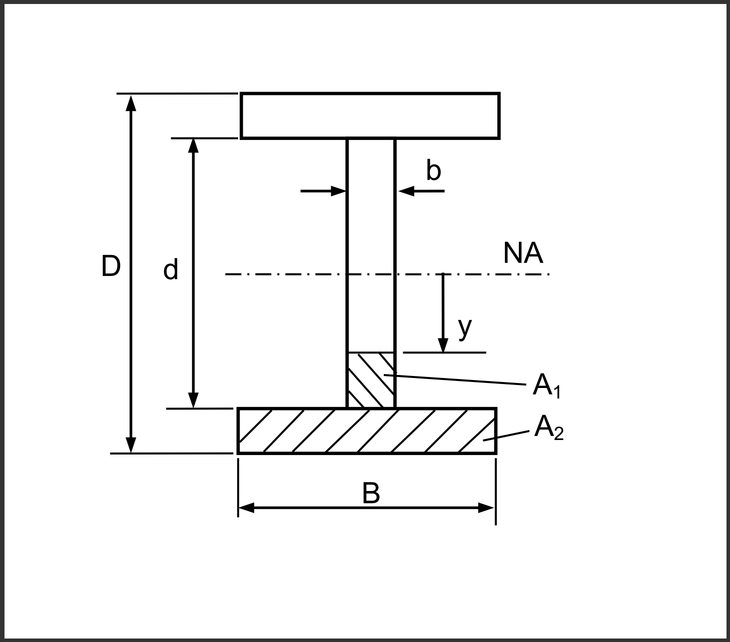
Shear Stress Distribution in an I-Beam (5)

- In the flange:

$$\tau = \frac{S}{Iz} A\bar{y}$$

$$\tau = \frac{S}{Iz}\left[B\left(\frac{D}{2} - y\right)\frac{1}{2}\left(\frac{D}{2} + y\right)\right]$$

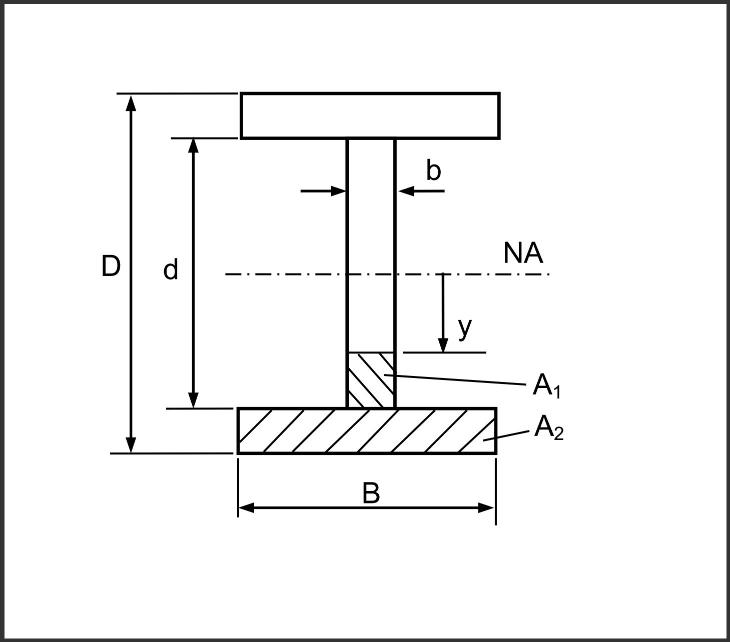
$$\tau = \frac{S}{2I}\left(\frac{D^2}{4} - y^2\right)$$



Shear Stress Distribution in an I-Beam (6)

- At y = +/- D/2, τ = 0

$$\tau = \frac{S}{2I}\left(\frac{D^2}{4} - y^2\right)$$



- And at y = +/- d/2

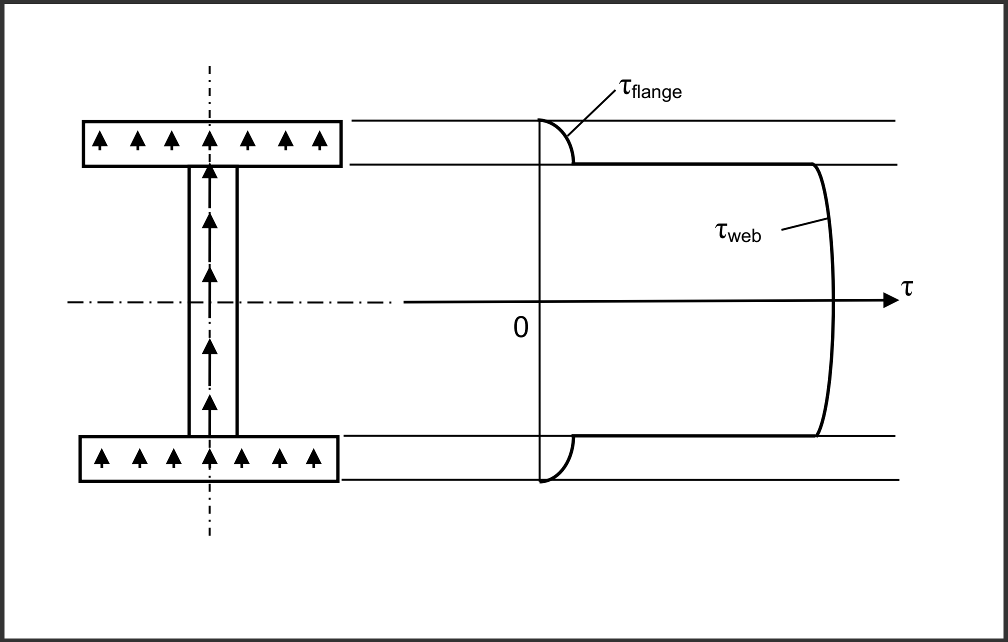
$$\tau = \frac{S}{8I}(D^2 - d^2)$$

- Compare with previous expression at this location:
- Step change in τ at this location due to change in section width

$$\tau = \frac{S}{Ib}\frac{B}{8}(D^2 - d^2)$$

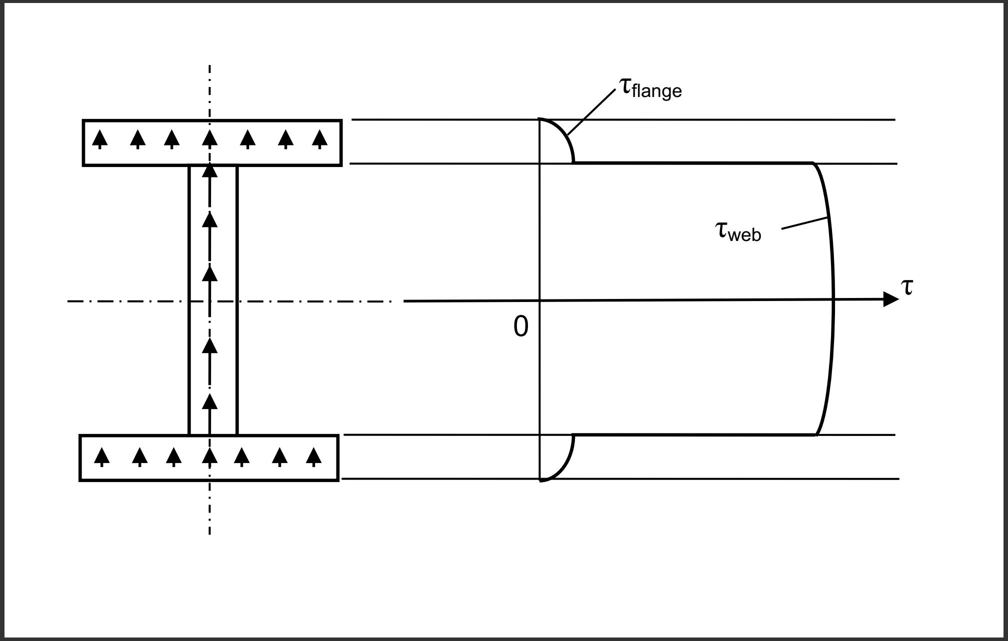
Shear Stress Distribution in an I-Beam (7)

- The shear stress distribution down the centre line of the section is shown and illustrates the step change at the junction of the web and the flange



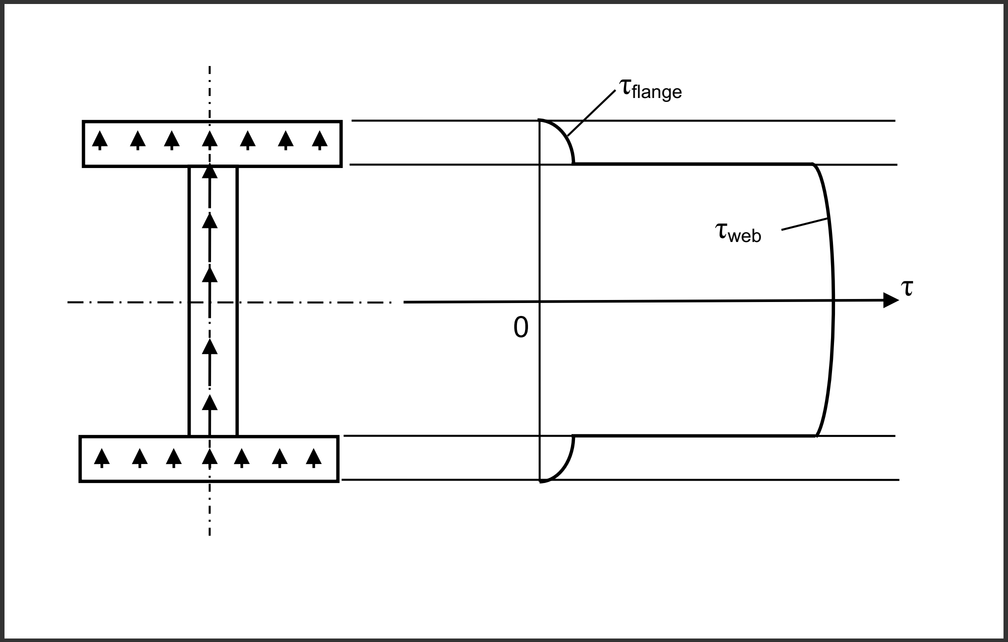
Shear Stress Distribution in an I-Beam (8)

- The shear in the flanges is small compared to the web where it is approximately uniform with vertical position



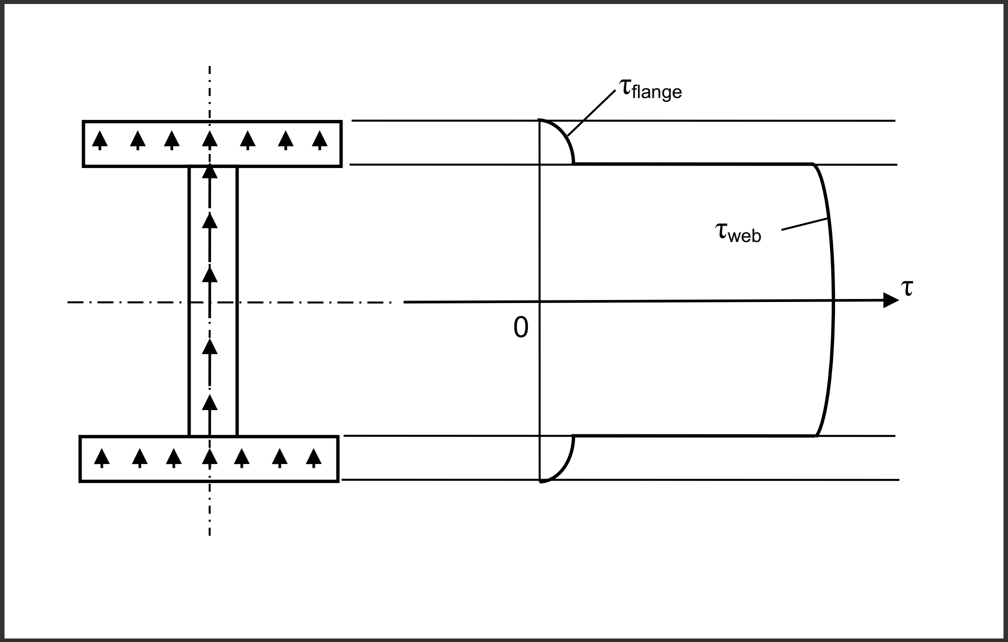
Shear Stress Distribution in an I-Beam (9)

- Because of the small shear in the flanges, the average shear stress in the web is ≈ S / bd i.e. the shear force divided by the area of the web



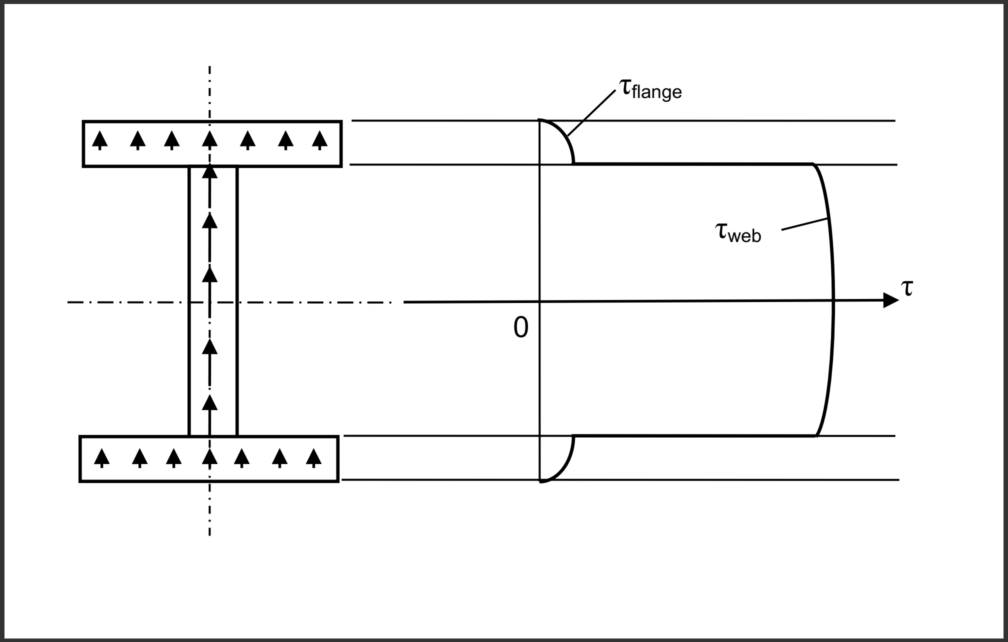
Shear Stress Distribution in an I-Beam (10)

- This distribution only applies down the centre line of the web.



Shear Stress Distribution in an I-Beam (11)

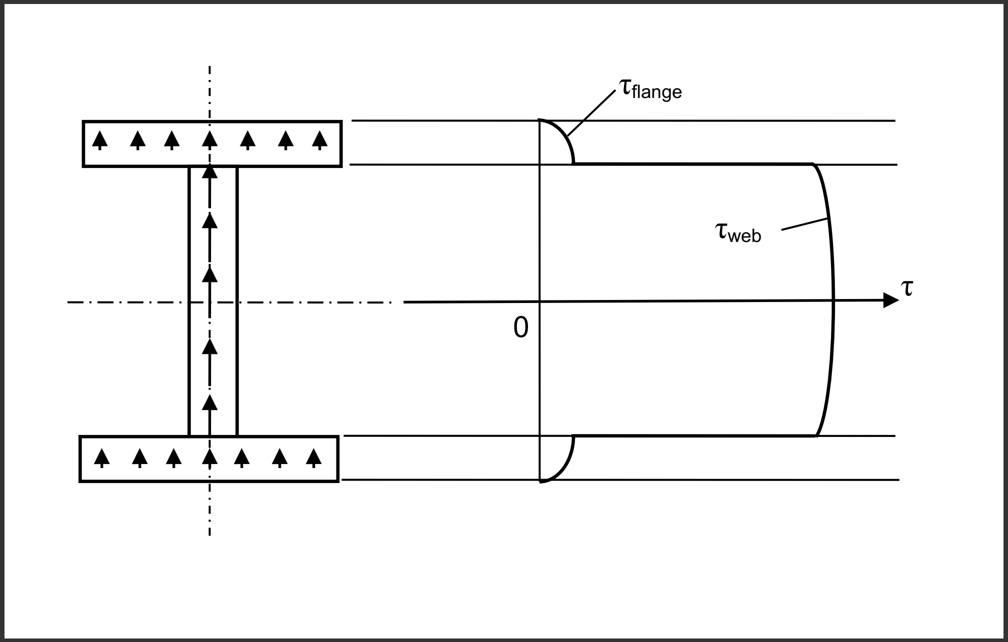
- The shear stresses in the flanges are small and non-uniform across the width. This must be the case as they must be zero at the top and bottom surfaces (i.e. free surfaces) of the flanges.



Shear Stress Distribution in an I-Beam (12)

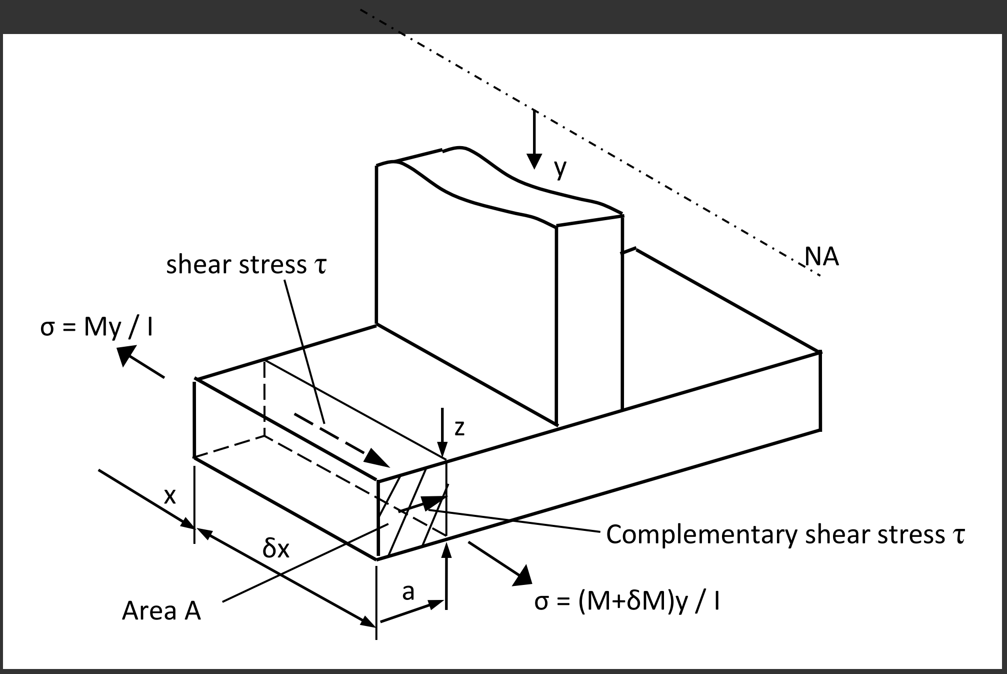
- There are more significant shear stresses in the flanges which act parallel to the flanges i.e. horizontally

- We will address these next…



Shear Stress Distribution in an I-Beam (13)

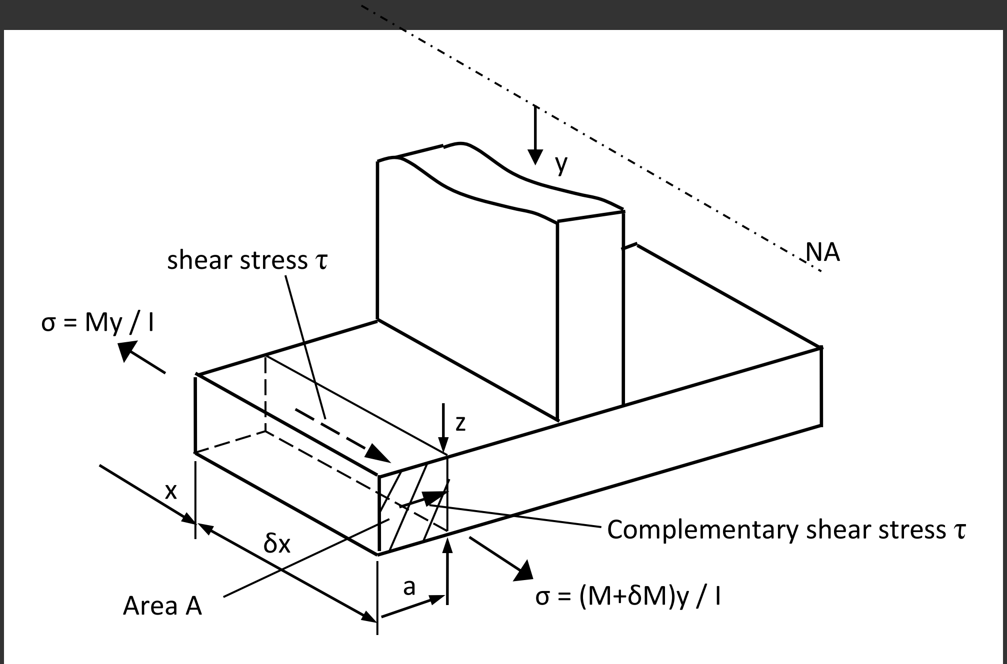
- Horizontal shear stress in the flanges



- To determine the horizontal shear stress, τ, at distance a from the edge of the flange, equilibrium of an element of the flange is considered

Shear Stress Distribution in an I-Beam (14)

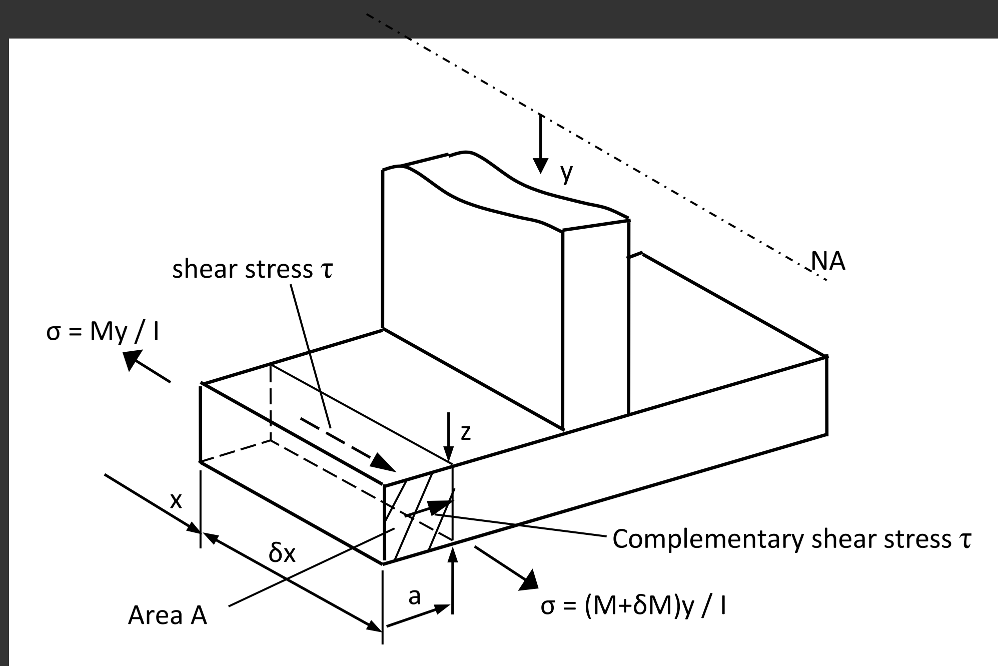
- Equilibrium of stresses acting on the element gives:



$$\int_A \frac{(M+\delta M)}{I} y dA - \int_A \frac{M}{I} y dA + \tau x \delta x = 0$$

Shear Stress Distribution in an I-Beam (15)

- Equilibrium of stresses acting on the element gives:



$$\tau = -\frac{1}{Iz}\frac{\delta M}{\delta x}\int_A y dA$$

Shear Stress Distribution in an I-Beam (16)

- In the limit

$$\frac{\lim}{\delta x \to 0} \frac{\delta M}{\delta x} = \frac{dM}{dx} = -S$$

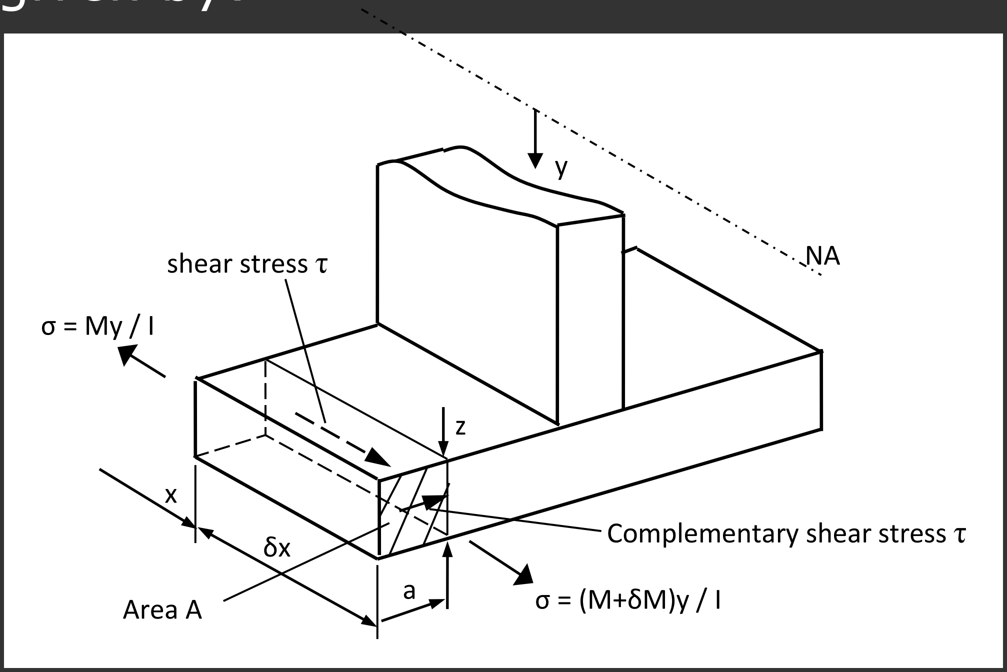
- Which gives:

$$\tau = \frac{S}{Iz} \int_A ydA = \frac{S}{Iz} A\bar{y}$$

- The same expression as for the vertical shear stress – handy!

Shear Stress Distribution in an I-Beam (17)

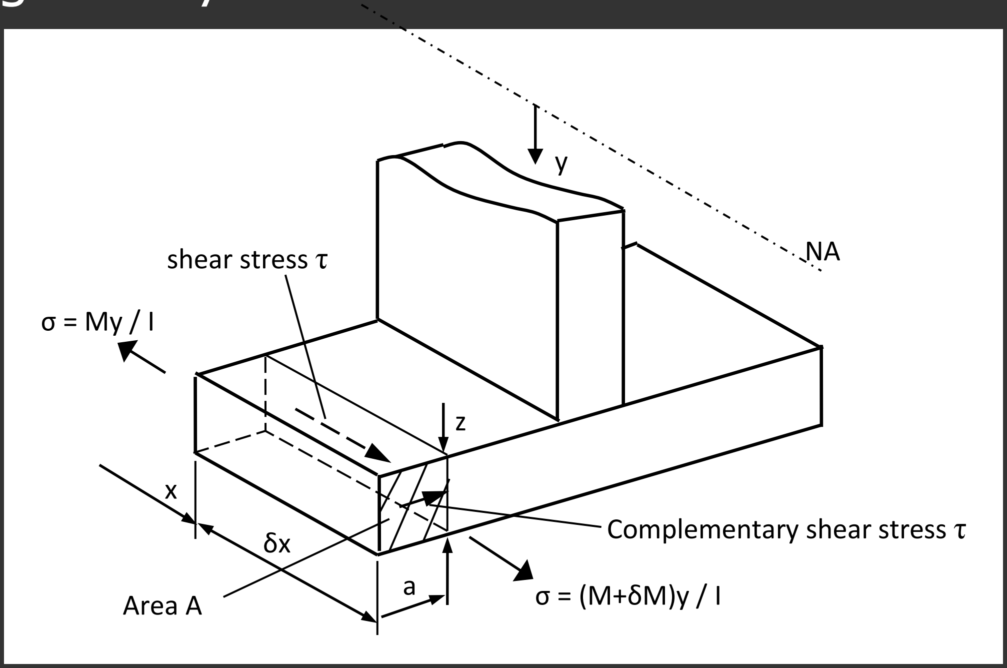
- A, $\bar{y}$ and z are slightly different as shown below. At a distance a from the edge of the flange, the horizontal shear stress is given by:



$$\tau = \frac{S}{Iz}(az)\frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right)$$

Shear Stress Distribution in an I-Beam (18)

- A, $\bar{y}$ and z are slightly different as shown below. At a distance a from the edge of the flange, the horizontal shear stress is given by:



$$\tau = \frac{Sa}{4I}(D + d)$$

Shear Stress Distribution in an I-Beam (19)

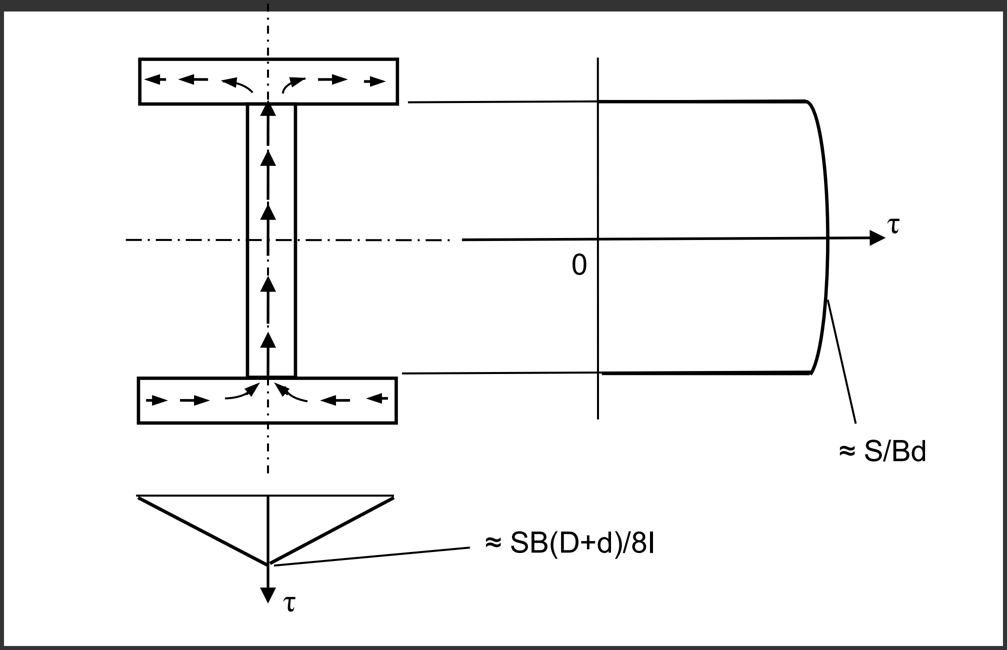
- τ therefore varies linearly with a from zero at the flange edge to a maximum value at the flange centre (a=B/2):

$$\tau_{max} = \frac{SB}{8I}(D + d)$$

- τ is also parallel to the flange i.e. horizontal.

Shear Stress Distribution in an I-Beam (20)

- We can now draw the dominant shear stresses in both the flange and the web



- The critical stress position is likely to be at the join of the web and flange where both the shear and bending stresses are high

Learning Objectives

1. Appreciate that in addition to longitudinal bending stresses, beams also carry transverse shear stresses arising from the vertical shear loads acting within the beam (knowledge)
2. Be able to derive a general formula, in both integral and discrete form, for evaluating the shear stress distribution through a cross-section (comprehension);
3. Determine the shear stress distribution through the thickness in a rectangular, circular and I-section beam (application);

Learning Objectives

4. Understand that in an I-section, in addition to the transverse vertical shear stresses in the flange and web, more dominant horizontal shear stresses also occur in the flange (comprehension);
5. Recognise that the resultant of the shear stresses always act through one point, known as the ‘shear centre’ (comprehension);
6. Calculate the position of the shear centre (application);
7. Understand that if the applied loads do not act through the shear centre, then there is a resultant torsional load, which can result in twisting of the section if the torsional rigidity of the section is low e.g. thin walled sections (comprehension).