



Fatigue & Fracture

Lecture 1 – Fatigue



Fatigue & Fracture

Introduction

Fatigue failure of components and structures results from cyclic (or repeated) loading and from the associated cyclic stresses & strains, as opposed to failure due to monotonic or static stresses & strains, such as buckling or plastic collapse due to excessive plastic deformation yielding.

The topic of fatigue is extremely important in mechanical engineering, since machines have moving parts, which in turn give rise to stresses & strains which may vary with time, typically in a repetitive fashion.

For example, the axle of a car which will transmit a time-varying torque, that changes from zero to some finite value when the car is put into gear and driven (and back to zero again when the car is taken out of gear).

An important design consideration, with respect to fatigue, is the fact that fatigue failure can occur at stresses which are well below the ultimate tensile strength of the material and often below the yield strength.

Fracture is concerned with the failure (rupture) of materials due to fatigue loading.

Fatigue & Fracture

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1. Know the various stages leading to fatigue failure (knowledge);
2. Know the basis of the total life approach and of the damage tolerant approach to estimate the number of cycles to failure (knowledge);
3. Be able to include the effects of mean and alternating stress on cycles to failure using the Gerber, modified Goodman and Soderberg methods (application);
4. Be able to include the effect of a stress concentration on fatigue life (application);
5. Be able to apply the S-N design procedure for fatigue life (application);
6. Know the meaning of linear-elastic fracture mechanics (LEFM) (knowledge);
7. Know what the three crack tip loading modes are (knowledge);
8. Know the meaning of fracture toughness (knowledge);
9. Understand the Paris equation for fatigue crack growth and the effects of the mean and alternating components of the stress intensity factor (knowledge/comprehension).

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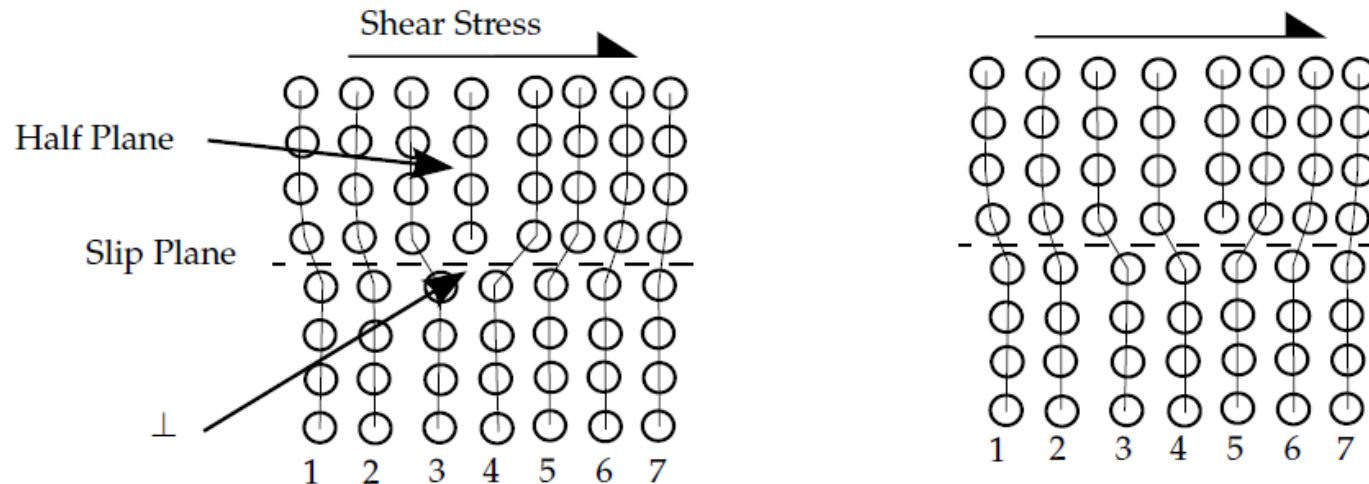
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Fatigue

Stage I Fatigue Failure: Crack Initiation

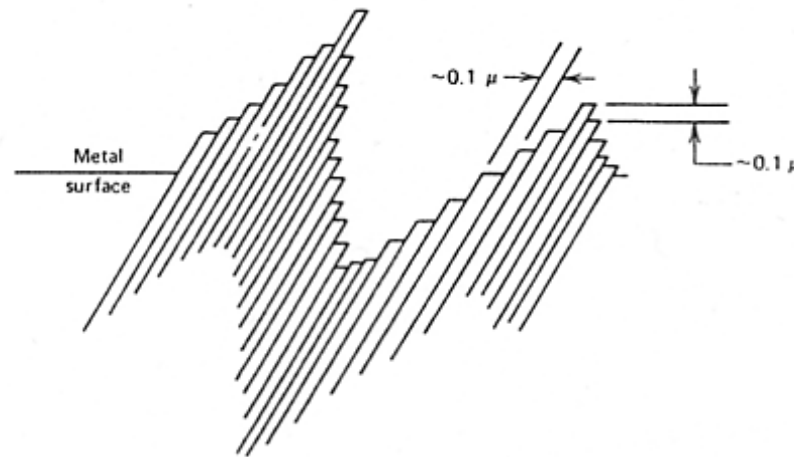
The micro-structural phenomenon which causes the initiation of fatigue cracks is the development of persistent slip bands at the surfaces of the specimen.

These persistent slip bands are the result of dislocations moving along crystallographic planes leading to both slip band intrusions and extrusions on the surface. These act as excellent stress concentrations and can thus lead to crack initiation.



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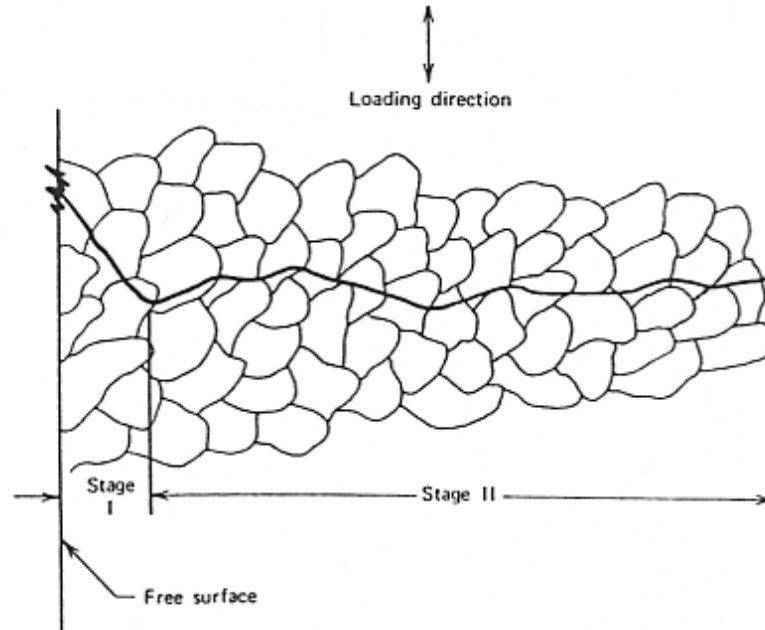
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Crystallographic slip is primarily controlled by shear stresses rather than normal stresses so that fatigue cracks initially tend to grow in a plane of maximum shear stress range. This stage leads to short cracks, usually only of the order of a few grains.

Stage II Fatigue Failure: Crack Propagation

As cycling continues, the fatigue cracks tend to coalesce and grow along planes of maximum tensile stress range.



Stage III Fatigue Failure: Final Fracture

Final fracture occurs when the crack reaches a critical length and results in either ductile tearing (plastic collapse) at one extreme, or cleavage (brittle fracture) at the other extreme.

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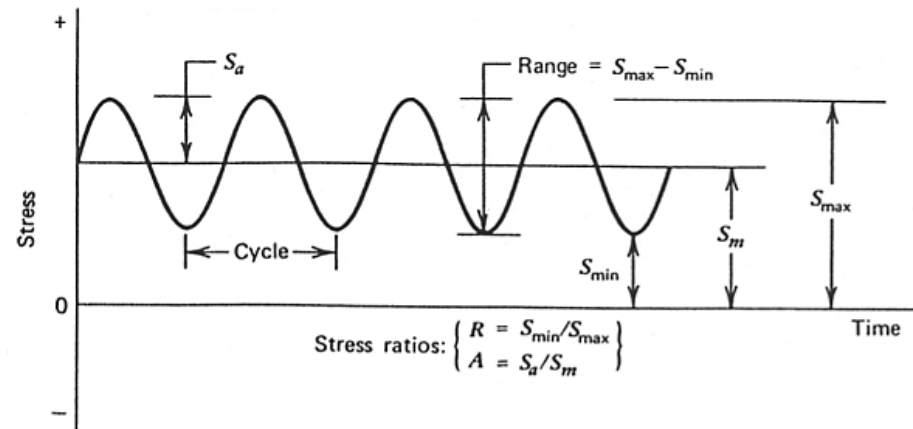
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Total Life Approach

The Total Life Approach is one of the established methods for the analysis and design of components which may be subject to fatigue conditions.

The Total Life Approach is based on laboratory tests, which are carried out under either stress- or strain-controlled loading conditions on idealised specimens, in order to obtain the numbers of cycles to failure as a function of the applied alternating stress (or strain).



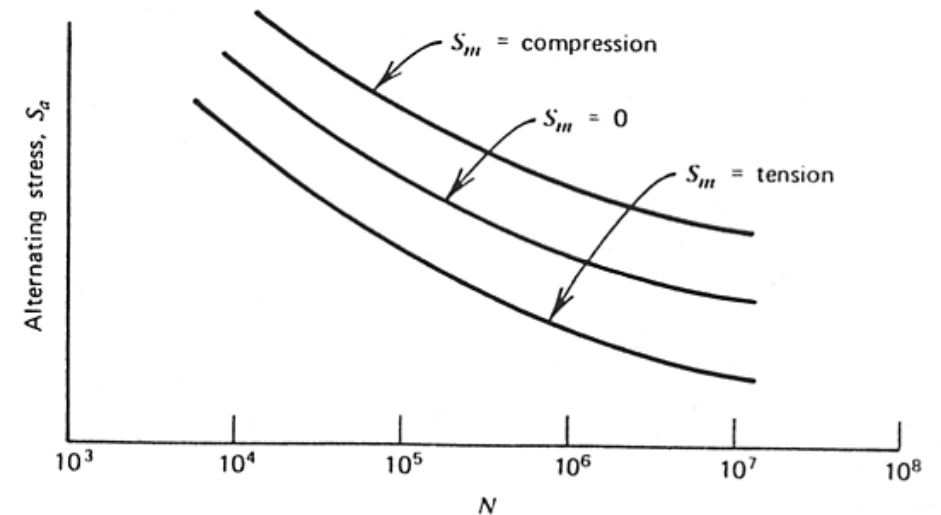
The specimens usually have finely polished surfaces to minimise surface roughness effects, which would particularly affect Stage I growth. In this approach, no distinction is made between crack initiation and propagation.

These tests furnish the number of loading cycles to the initiation of a 'measurable' crack as a function of applied stress or strain parameters.

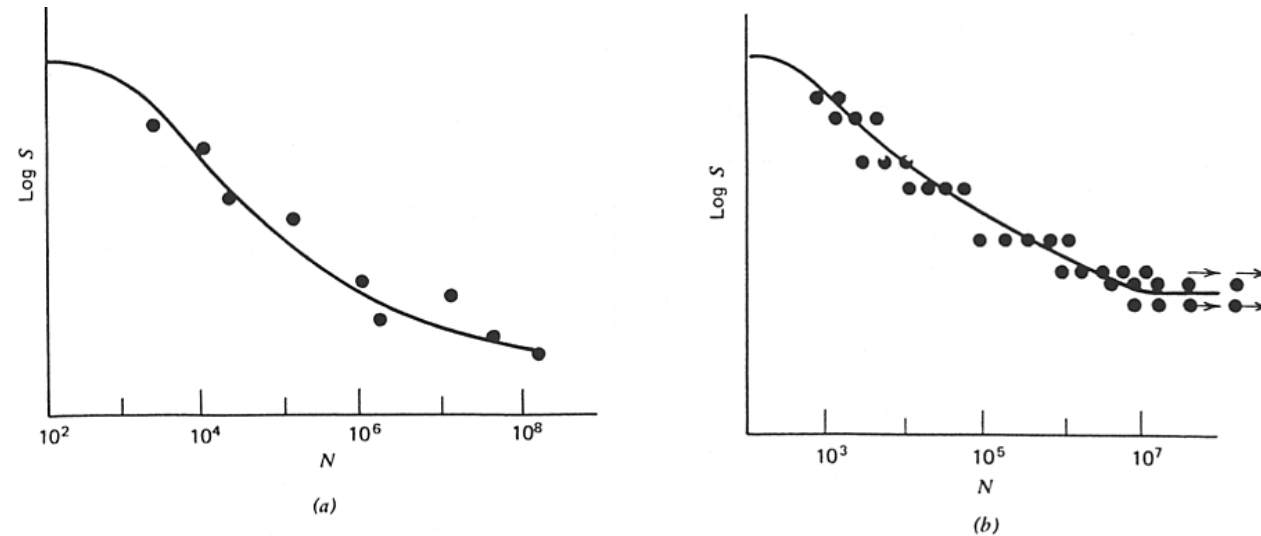
The 'measurability' is dictated by the resolution accuracy of the crack detection method employed. A typical 'measurable' crack is about 0.75 mm to 1 mm.

The challenge of fatigue design is then to relate these test results to actual component lives under in-service loading conditions.

Traditionally, most fatigue testing was based on fully-reversed (i.e. zero mean stress, $S_m = 0$), stress-controlled conditions and the fatigue design data was presented in the form of S - N curves, which are either semi-log or log-log plots of alternating stress, S_a , against the measured number of cycles to failure, N , where failure is defined as fracture.



The figure below contains schematic representations of two typical $S-N$ curves obtained from load (or stress)-controlled tests on smooth specimens where part a shows a continuously sloping curve, while part b shows a discontinuity or “knee” in the curve, which is only found in a few materials (notably low strength steels) between 10^6 and 10^7 cycles under non-corrosive conditions.

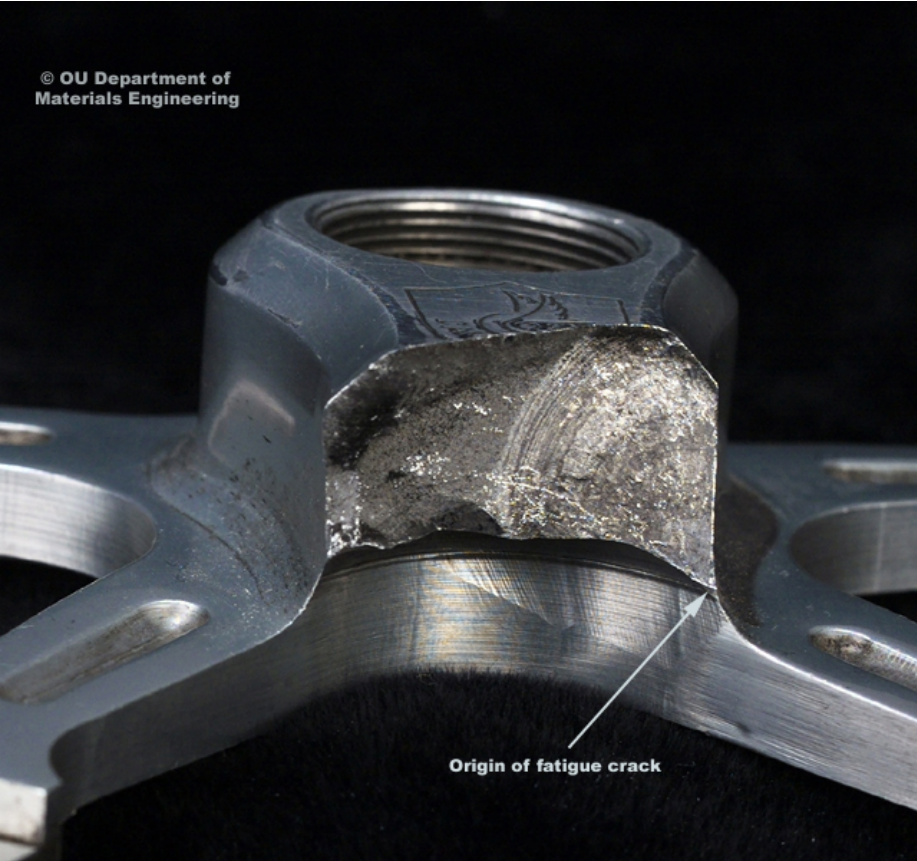


The curves are normally drawn through the median life value (of the scatter in N) and thus represents 50 percent expected failure.

Fatigue strength, S_e , is a hypothetical value of stress range at failure for exactly N cycles as obtained from an $S-N$ curve.

The fatigue limit (sometimes called the endurance limit) is the limiting value of the median fatigue strength as N becomes very large, e.g. $>10^8$ cycles.

Fatigue – Bicycle Crank Arm Failure Example

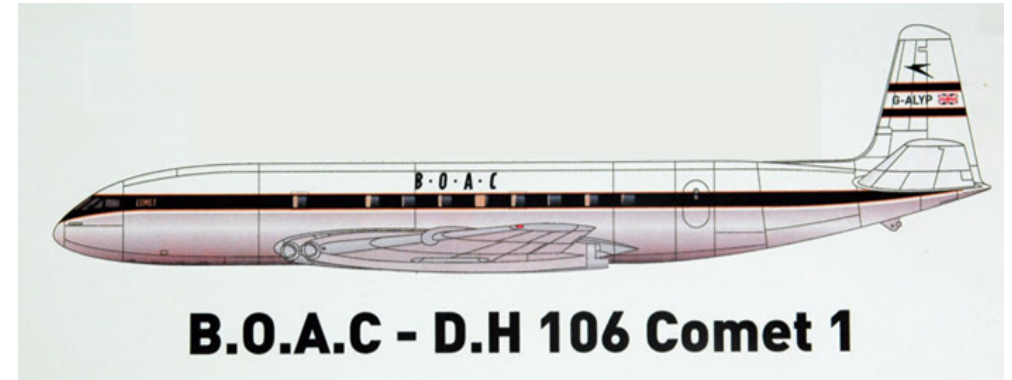


Fatigue – D.H.-106 Comet Failure Example

1st production jet liner (commercial debut 1952).

Several air crashes in 1954 led to an inquiry.

Water tank testing and examination of the recovered fuselage showed that failure originated at a square corner window.



Future designs incorporated an oval window

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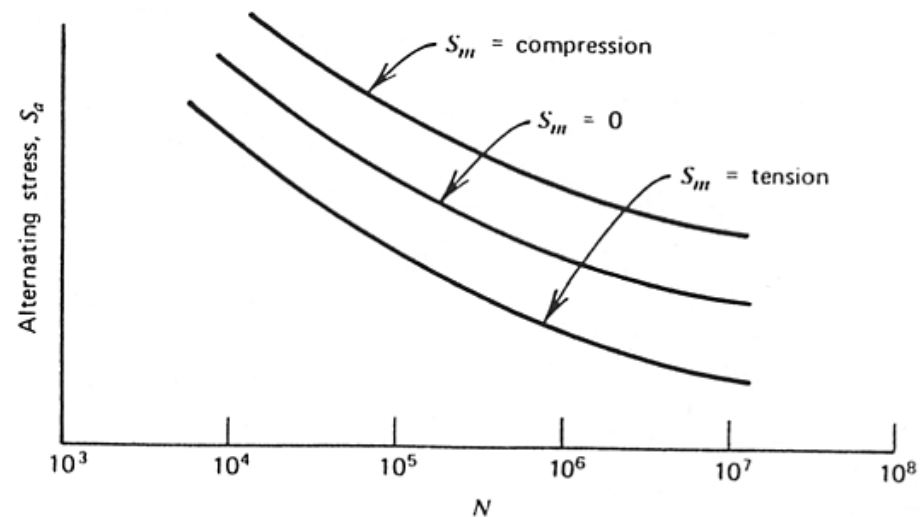
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Effect of mean stress

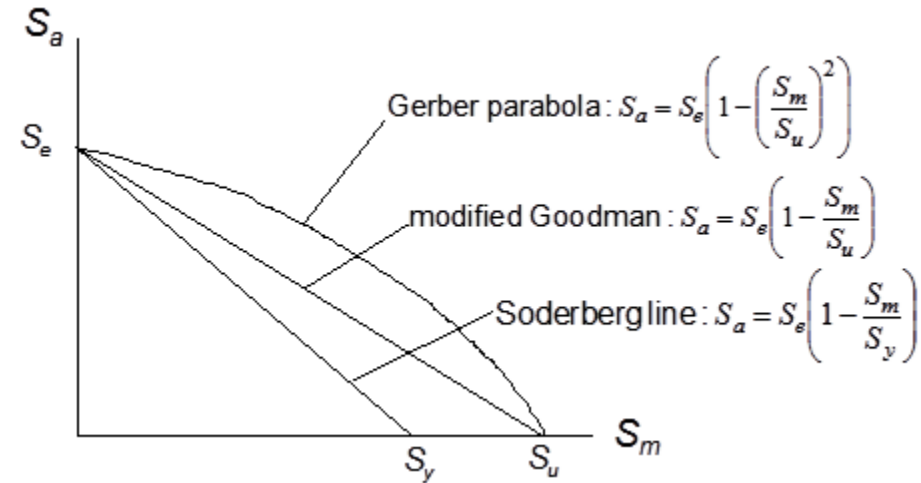
Early investigators of fatigue assumed that only the alternating stress affected the fatigue life of a cyclically-loaded component.

However, it has since been established that the mean stress has a significant effect on fatigue behaviour, as shown in the figure below.



It can be seen that tensile mean stresses are detrimental to fatigue life while compressive mean stresses are beneficial to fatigue life in comparison to zero mean stresses.

The effect of mean stress is commonly represented as a plot of S_a versus S_m for a given fatigue life. Attempts have been made to generalise this relationship. Three of these common relationships are shown in the figure below.



- The modified Goodman line assumes a linear relationship between the allowable S_a and the corresponding mean stress S_m , where the slope and intercepts are defined by the fatigue life, S_e , and the material UTS, S_u , respectively.
- The Gerber parabola employs the same end-points but, in this case, the relation is defined by a parabola.
- Finally, the Soderberg line again assumes a linear relation, but this time the mean stress axis end-point is taken as the yield stress, S_y .

These curves can be extended into the compressive mean stress region to give increasing allowable alternating stress with increasing compressive mean stress, but this is normally taken to be horizontal for design purposes and for conservatism.

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Effect of stress concentrations

Since the first occurrences of fatigue failure, it has been recognised that such failures are most commonly associated with notch-type features in components.

The stress concentration associated with notch-type features leads typically to local plastic strain which eventually leads to fatigue cracking.

Consequently, the estimation of stress concentration factors associated with various types of notches and geometrical discontinuities has received a lot of attention. This is typically expressed in terms of an elastic stress concentration factor (SCF), K_t , which is simply the relationship between the maximum local stress and an appropriate nominal stress, as follows:

$$K_t = \frac{\sigma_{\max}^{\text{el}}}{\sigma_{\text{nom}}}$$

It was once thought that the fatigue strength of a notched component could be predicted as the strength of a smooth component divided by the SCF.

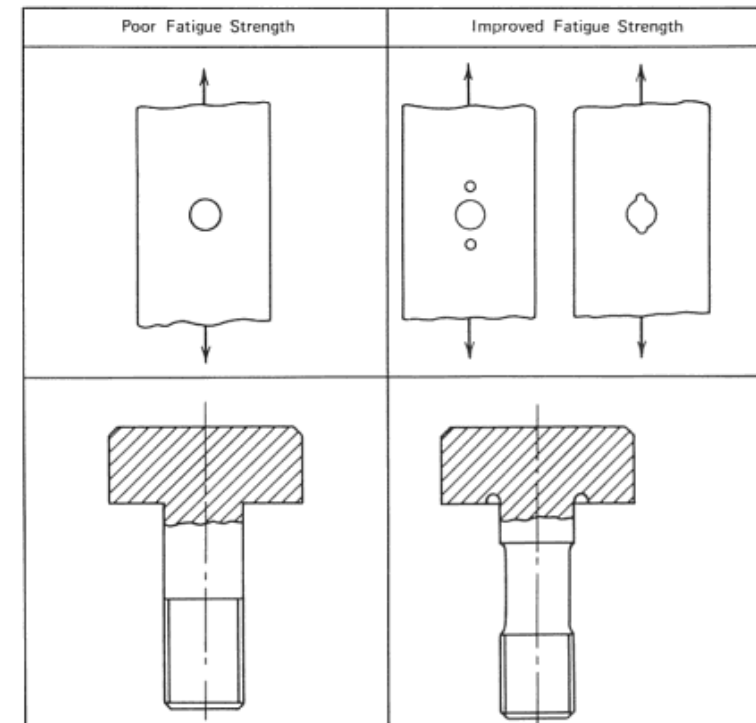
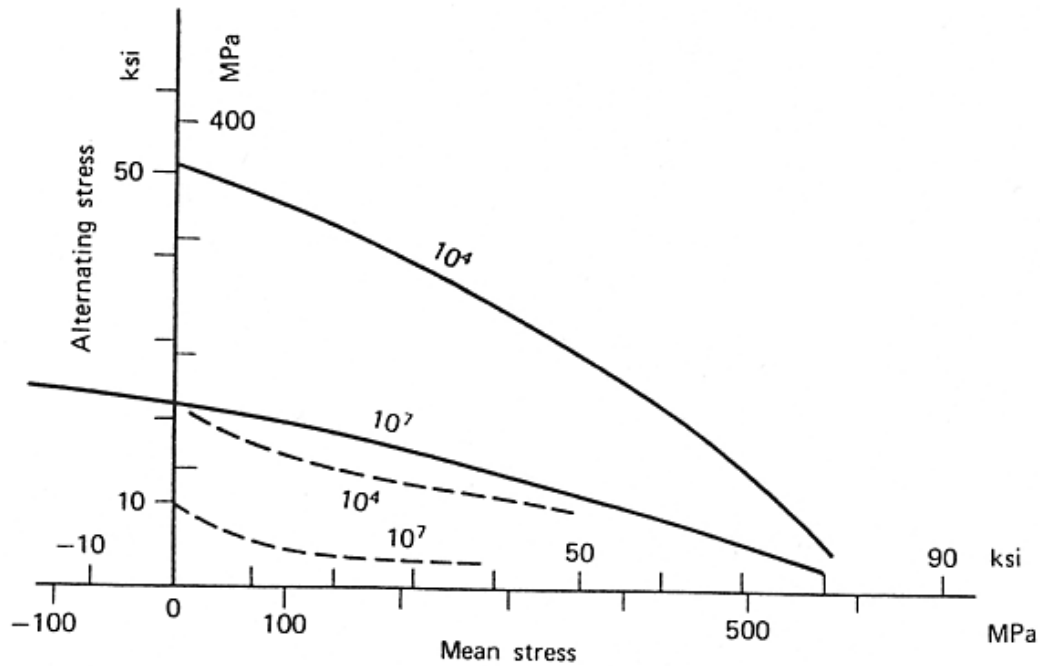
However, this is not the case. The reduction is, in fact, often less than K_t and is defined by the fatigue notch factor, K_f , which is defined as the ratio of the smooth fatigue strength to the notched fatigue strength as follows:

$$K_f = \frac{S_a^{\text{smooth}}}{S_a^{\text{notch}}}$$

However, this fatigue notch factor is also found to vary with both alternating and mean stress level and thus with number of cycles to failure.

The left-hand figure below shows the effect of a notch, with an SCF of 3.4, on the fatigue behaviour of a wrought aluminium alloy, where the smooth lines are for the smooth specimen and the dotted lines are for the notched specimen.

Examples of components with poor and improved fatigue strength are shown in the right hand figure below.



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S-N Design Procedure for Fatigue

Constant life diagrams plotted as S_a versus S_m , also called Goodman diagrams, as shown in the figure below, can be used in design to give safe estimates of fatigue life and loads. The Goodman line connects the endurance limit, S_e (or long life fatigue strength), to the U.T.S., S_u .

The fatigue strength for zero mean stress is reduced by the fatigue notch factor, K_f . The stress concentration factor, K_t is used if K_f is not known.

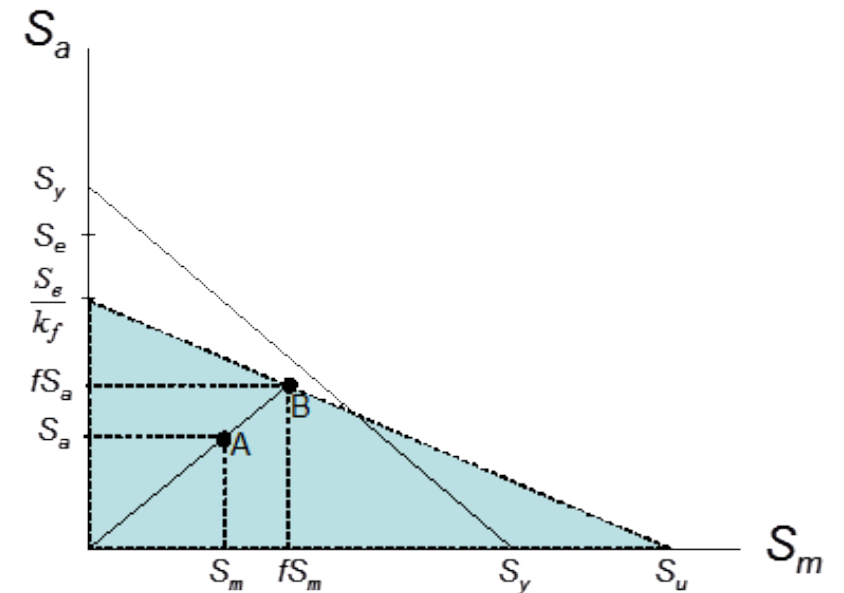
For static loading of a ductile component with a stress concentration, failure still occurs when the mean stress is equal to the U.T.S. Failure at intermediate values of mean stress is assumed to be given by the dotted line.

In order to avoid yield of the whole cross-section of the component, the maximum nominal stress must be less than the yield stress, S_y , i.e. $S_m + S_a < S_y$. This relationship gives the yield line joining S_y to S_y .

The factor of safety is determined from the position of the point relative to the safe/fail boundary. I.e., factor of safety $F = OB/OA$

From similar triangles:
$$\frac{S_a}{\left(\frac{S_u}{F} - S_m\right)} = \frac{S_e}{k_f S_u}$$

Therefore:
$$\frac{1}{F} = \frac{S_a k_f}{S_e} + \frac{S_m}{S_u}$$



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