Fatigue & Fracture

Lecture 2 – Fracture

Department of Mechanical, Materials & Manufacturing Engineering

MMME2053 – Mechanics of Solids

University of Nottingham
UK | CHINA | MALAYSIA

Fracture

Introduction

Cracks may initiate in components due to several mechanisms (for example, creep and/or fatigue).

These cracks can then propagate (grow), leading to catastrophic failure.

Analysis of components containing cracks is therefore an important aspect of design if safe operation is to be achieved.

In the present topic, it will be assumed that materials operate in the linear elastic region.

The objective is to determine when unstable crack growth (fracture) will occur.

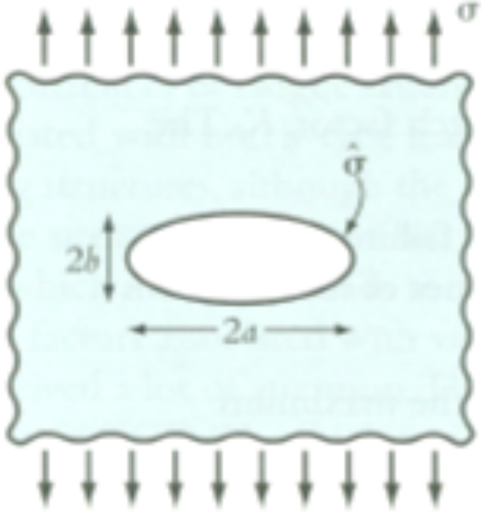
Fatigue & Fracture

Learning Outcomes

1. Know the various stages leading to fatigue failure (knowledge);
2. Know the basis of the total life approach and of the damage tolerant approach to estimate the number of cycles to failure (knowledge);
3. Be able to include the effects of mean and alternating stress on cycles to failure using the Gerber, modified Goodman and Soderberg methods (application);
4. Be able to include the effect of a stress concentration on fatigue life (application);
5. Be able to apply the S-N design procedure for fatigue life (application);
6. Know the meaning of linear-elastic fracture mechanics (LEFM) (knowledge);
7. Know what the three crack tip loading modes are (knowledge);
8. Know the meaning of fracture toughness (knowledge);
9. Understand the Paris equation for fatigue crack growth and the effects of the mean and alternating components of the stress intensity factor (knowledge/comprehension).

Fracture

Consider the stress concentration factor for an elliptical hole in a large, linear-elastic plate subjected to a remote, uniaxial stress.



It can be shown that stress concentration factor can be expressed as: $K_t = \frac{\sigma_{\max}^{\mathrm{el}}}{\sigma_{\mathrm{nom}}} = 1 + \frac{2a}{b}$

Thus, as b → 0, the elliptical hole degenerates to a crack, and $a/b \to \infty$, so that the notch stress also goes to infinity (i.e. becomes singular), $K_t \to \infty$, provided the material behaviour remains linear elastic.

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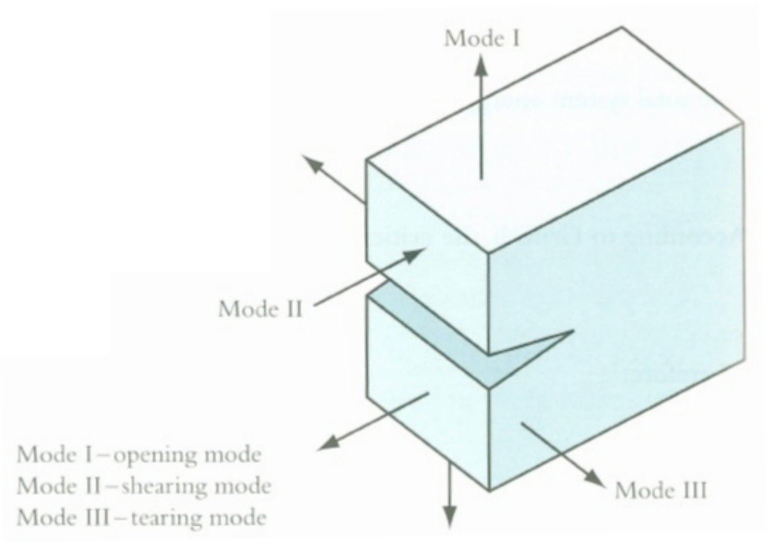
Basis of the energy approach to fracture mechanics

So far, we have only considered the so-called Mode I loading case. There are actually three different loading modes considered in fracture mechanics, as shown in the figure below.



Generally, for geometries with finite boundaries, the following expression is employed for stress intensity factor:

$$K_I = Y\sigma\sqrt{\pi a}$$

K_I is the Mode-I stress-intensity factor (units MPa√m) which defines the magnitude of the elastic stress field in the vicinity of the crack tip.

Similarly for K_{II} and K_{III}, where Y is a function of the crack and component dimensions.

In general, the energy release rate under mixed-mode loading is given by:

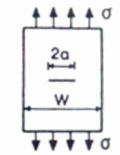
$$K_{total} = K_I + K_{II} + K_{III}$$

Fracture Toughness

Typical Fracture Toughness Values

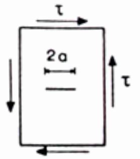
Material	$\sigma_y \left(MN/m^2\right)$	$K_{Ic}\left(MN/m^{3/2}\right)$
Mild Steel	220	140 to 200
Pressure Vessel Steel (HY130)	1700	170
Aluminium Alloys	100 to 600	45 to 23
Cast Iron	200 to 1000	20 to 6

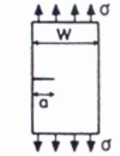
The Effect of Finite Boundaries on Expressions for Stress Intensity Factors



$$K_I = \sigma\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}$$

$$K_{II} = \tau\sqrt{\pi a}\left(\text{small } \frac{a}{W}\right)$$



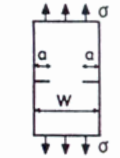


$$K_I = 1.12\,\sigma\sqrt{\pi a}\left(\text{small } \frac{a}{W}\right)$$

or $K_I = Y\sigma\sqrt{a}$

with $Y = 1.99 - 0.41\dfrac{a}{W} + 18.7\left(\dfrac{a}{W}\right)^2 - 38.48\left(\dfrac{a}{W}\right)^3 + 53.85\left(\dfrac{a}{W}\right)^4$

$(1.99 = 1.12\sqrt{\pi})$

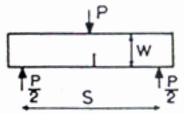


$$K_I = 1.12\,\sigma\sqrt{\pi a}\left(\text{small } \frac{a}{W}\right)$$

or $K_I = Y\sigma\sqrt{a}$

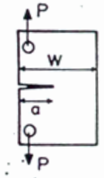
with $Y = 1.99 + 0.76\dfrac{a}{W} - 8.48\left(\dfrac{a}{W}\right)^2 + 27.36\left(\dfrac{a}{W}\right)^3$

$(1.99 = 1.12\sqrt{\pi})$



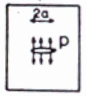
Thickness B

$$K_I = \frac{PS}{BW^{3/2}}\left[2.9\left(\frac{a}{W}\right)^{1/2} - 4.6\left(\frac{a}{W}\right)^{3/2} + 21.8\left(\frac{a}{W}\right)^{5/2} - 37.6\left(\frac{a}{W}\right)^{7/2} + 38.7\left(\frac{a}{W}\right)^{9/2}\right]$$



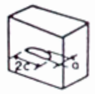
Thickness B

$$K_I = \frac{P}{BW^{1/2}}\left[29.6\left(\frac{a}{W}\right)^{1/2} - 185.5\left(\frac{a}{W}\right)^{3/2} + 655.7\left(\frac{a}{W}\right)^{5/2} - 1017\left(\frac{a}{W}\right)^{7/2} + 63.9\left(\frac{a}{W}\right)^{9/2}\right]$$



p per unit thickness

$K_I = p\sqrt{\pi a}$



$$K_{I_{max}} = 1.12\frac{\sigma}{\Phi}\sqrt{\pi a}$$

$$K_{I_{min}} = 1.12\frac{\sigma}{\Phi}\sqrt{\pi a^2/c}$$

$$\Phi = \int_0^{\pi/2}\left[1 - \frac{c^2 - a^2}{c^2}\sin^2\varphi\right]d\varphi$$

$$\Phi \approx \frac{3\pi}{8} + \frac{\pi}{8}\frac{a^2}{c^2}$$

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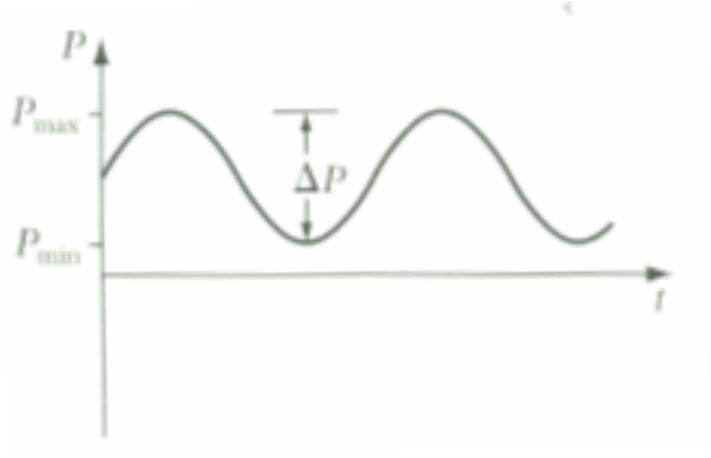
Fatigue Crack Growth

It has been shown by Paris and co-workers (1961) that, for a wide range of conditions, there is a logarithmic linear relationship between crack growth rate and the stress intensity factor range during cyclic loading of cracked components.

Although this proposition had difficulty being accepted initially, it has become the basis of the damage tolerant approach to fatigue life estimation and is widely used both in industry and in research.

Essentially, it means that crack growth can be modelled and estimated based on knowledge of crack and component geometry, loading conditions and using experimentally-measured crack growth data to furnish material constants.

Considering a load cycle as shown in the figure below which gives rise to a load range acting on a cracked body: $\Delta P = P_{\max} - P_{\min}$



The load range and crack geometry gives rise to a cyclic variation in stress intensity factor, which is given by: $\Delta K = K_{\max} - K_{\min}$

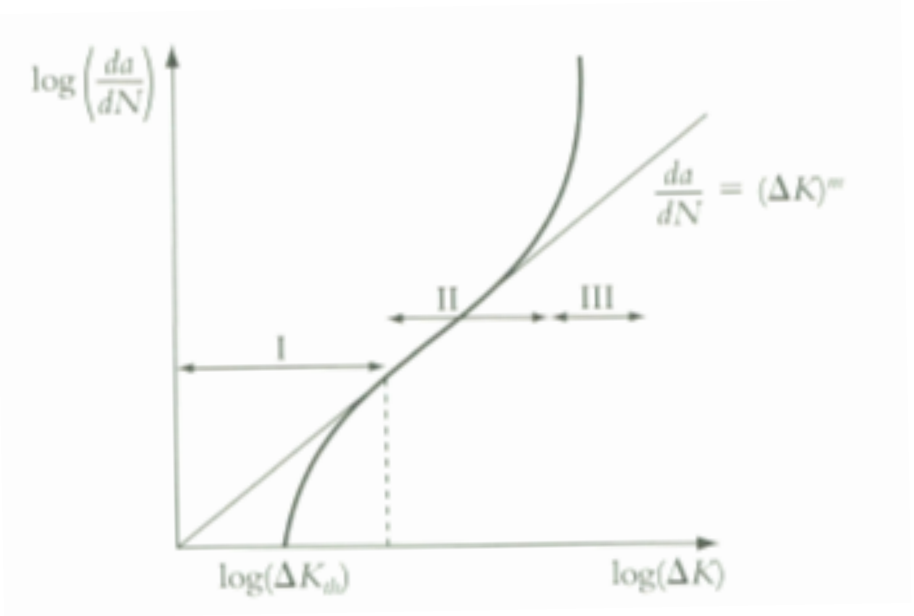
In addition, Paris showed that the subsequent crack growth can be represented by an empirical relationship as follows:

$$\frac{da}{dN} = C\Delta K^m$$

where C and m are empirically-determined material constants.

Fatigue crack growth data is often plotted as the logarithm of crack growth per load cycle, da/dN, and the logarithm of stress intensity factor range, for which there are three stages, as shown in the figure below.

Below ΔKth, no observable crack growth occurs; region II shows an essentially linear relationship between log(da/dN) and log(ΔK), where m is the slope of the curve and C is the vertical axis intercept; in region III, rapid crack growth occurs and little life is involved.



The linear regime (Region II) is the region in which engineering components which fail by fatigue propagation occupy most of their life.

Knowing the stress intensity factor expression for a given component and loading, the fatigue crack growth life of the component can be obtained by integrating the Paris Equation between the limits of initial crack size and final crack size.

This is achieved by taking logs of both sides of the Paris equation to give: $\log\left(\frac{da}{dN}\right) = m\log(\Delta K) + \log(C)$

This equation has the form of $y = mx + c$ and can be seen to represent region II of the plot above.

Typical values for ΔK_{th}, m and ΔK

Material	ΔK_{th} ($MN/m^{3/2}$)	m	ΔK ($MN/m^{3/2}$) for da/dN = 10^{-6} mm/cycle
Mild Steel	4 to 7	3.3	6.2
316 stainless steel	4 to 6	3.1	6.3
Aluminium	1 to 2	2.9	2.9
Copper	1 to 3	3.9	4.3
Brass	2 to 4	4.0	4.3 to 66.3
Nickel	4 to 8	4.0	8.8

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