Asymmetrical Bending

Worked Example 2 – Stresses in an Asymmetrically Loaded Beam

Department of Mechanical, Materials & Manufacturing Engineering

MMME2053 – Mechanics of Solids

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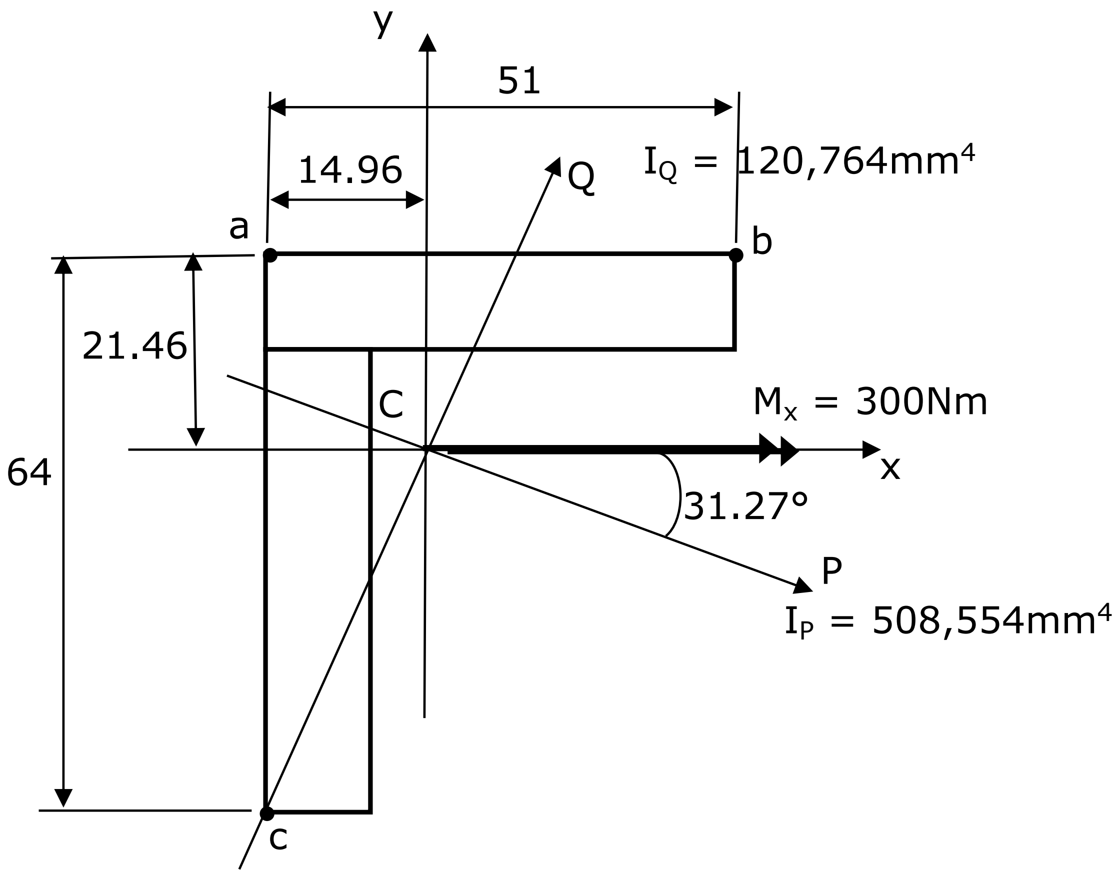
Worked Example 2

Stresses in an Asymmetrically Loaded Beam

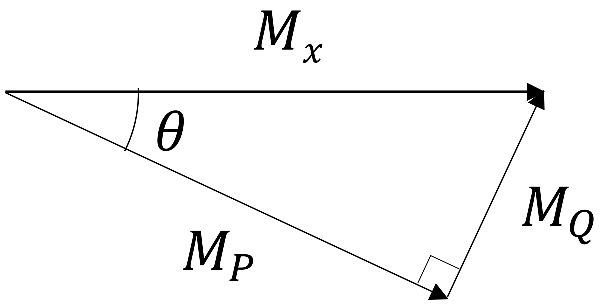
For the beam cross-section shown in worked example 1, a bending moment, $M_x = 300\ \mathrm{Nm}$ is applied along the x axis as shown, calculate:

- the position/orientation of the neutral axis
- the bending stresses at positions a, b and c



Solution

Resolve applied bending moment onto principal axes



$$\cos\theta = \frac{M_P}{M_x}$$

$$\therefore M_P = M_x \cos\theta$$

$$= 300 \times 10^3 \times \cos(31.27)$$

$$\therefore M_P = 256{,}419.2 \text{ Nmm}$$

$$\sin\theta = \frac{M_Q}{M_x}$$

$$\therefore M_Q = M_x \sin\theta$$

$$= 300 \times 10^3 \times \sin(31.27)$$

$$\therefore M_Q = 155{,}721.5 \text{ Nmm}$$

Determine the Position of the Neutral Axis

At the neutral axis there is no stress. Therefore:

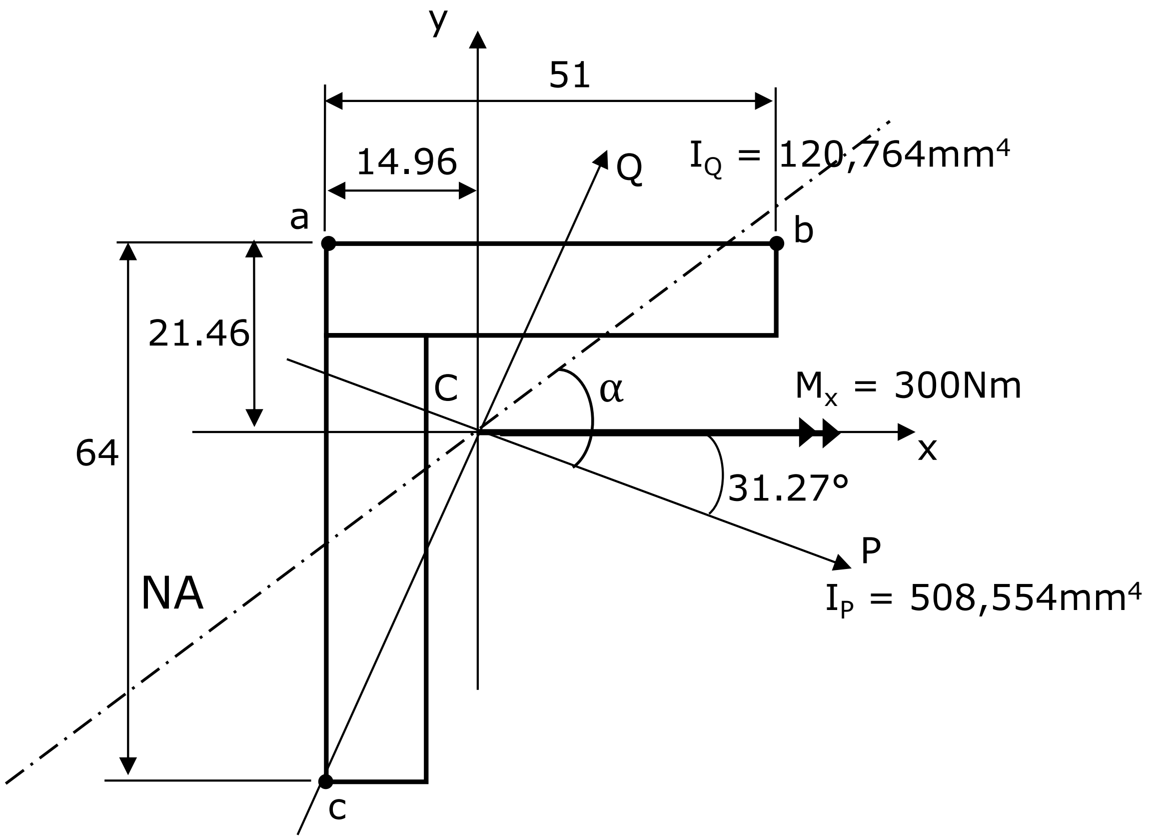
$$\sigma = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = 0$$

$$\therefore \frac{M_P Q}{I_P} = \frac{M_Q P}{I_Q}$$

$$\therefore \frac{Q}{P} = \frac{M_Q I_P}{M_P I_Q}$$

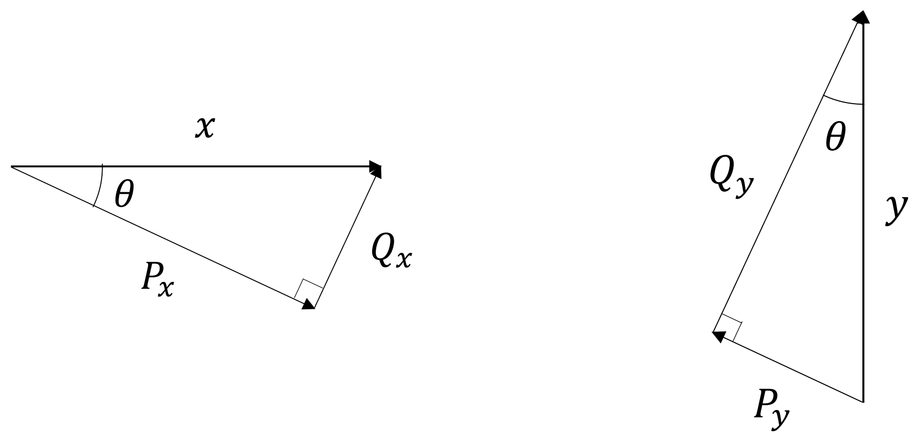
$$\therefore \alpha = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{M_Q I_P}{M_P I_Q}\right)\tan^{-1}\left(\frac{155{,}721.5 \times 508{,}554}{256{,}419.2 \times 120{,}764}\right)$$

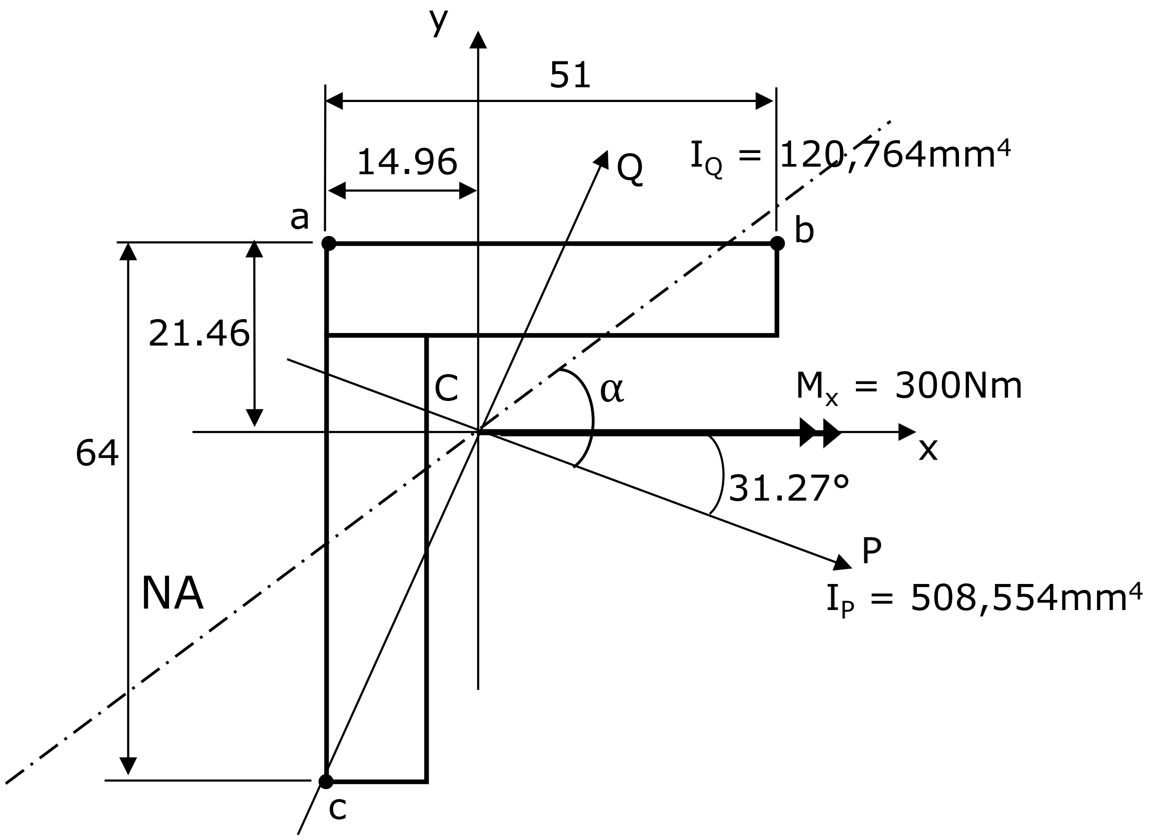
$$\therefore \alpha = 68.64°$$



Determine the Stresses at Positions a, b & c

Translation from x-y to P-Q axis:





$$P = P_x - P_y \quad \& \quad Q = Q_x + Q_y$$

$$\therefore P = x\cos\theta - y\sin\theta \quad \& \quad Q = x\sin\theta + y\cos\theta$$

At position a

$x = -14.96 \quad y = 21.46$

$\therefore P = x\cos\theta - y\sin\theta$

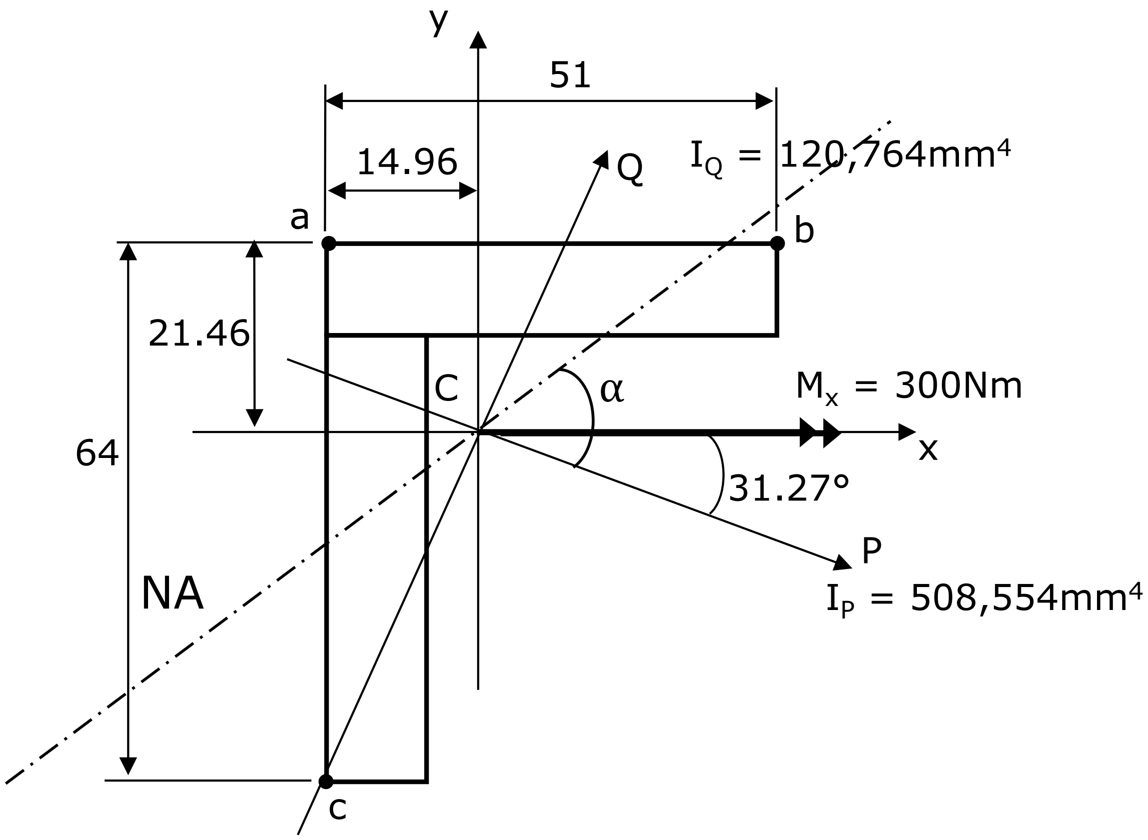
$= -14.96\cos(31.27) - 21.46\sin(31.27)$

$= -23.92$

$Q = x\sin\theta + y\cos\theta$

$= -14.96\sin(31.27) + 21.46\cos(31.27)$

$= 10.88$



$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{256{,}419.2 \times 10.88}{508{,}554} - \frac{155{,}721.5 \times -23.92}{120{,}764}$$

$\therefore \boldsymbol{\sigma_{b_a} = 36.33\ \mathrm{MPa}\ (\mathrm{tensile})}$

At position b

$x = 36.04 \qquad y = 21.46$

$\therefore P = x\cos\theta - y\sin\theta$

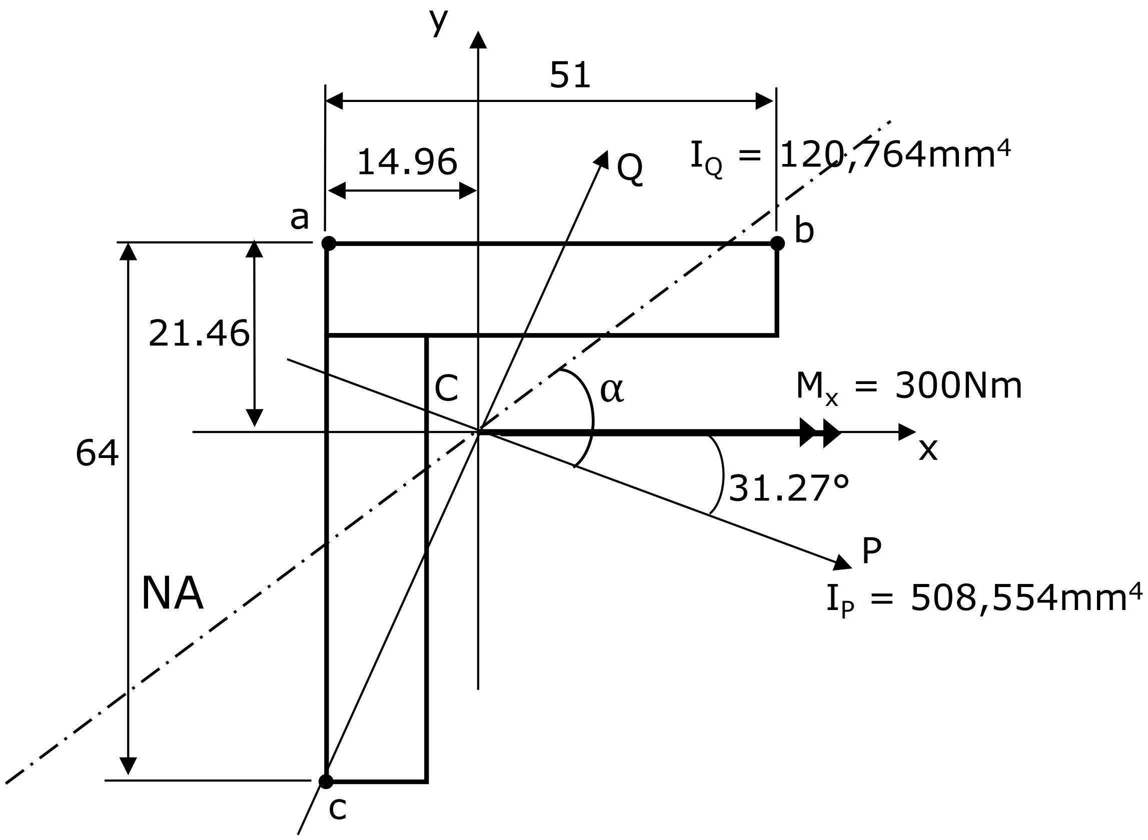
$= 36.04\cos(31.27) - 21.46\sin(31.27)$

$= 19.66$

$Q = x\sin\theta + y\cos\theta$

$= 36.04\sin(31.27) + 21.46\cos(31.27)$

$= 37.05$



$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{256{,}419.2 \times 37.05}{508{,}554} - \frac{155{,}721.5 \times 19.66}{120{,}764}$$

$\therefore \boldsymbol{\sigma_{b_b} = -6.67}$ **MPa (compressive)**

At position c

$x = -14.96 \qquad y = -42.54$

$\therefore P = x\cos\theta - y\sin\theta$

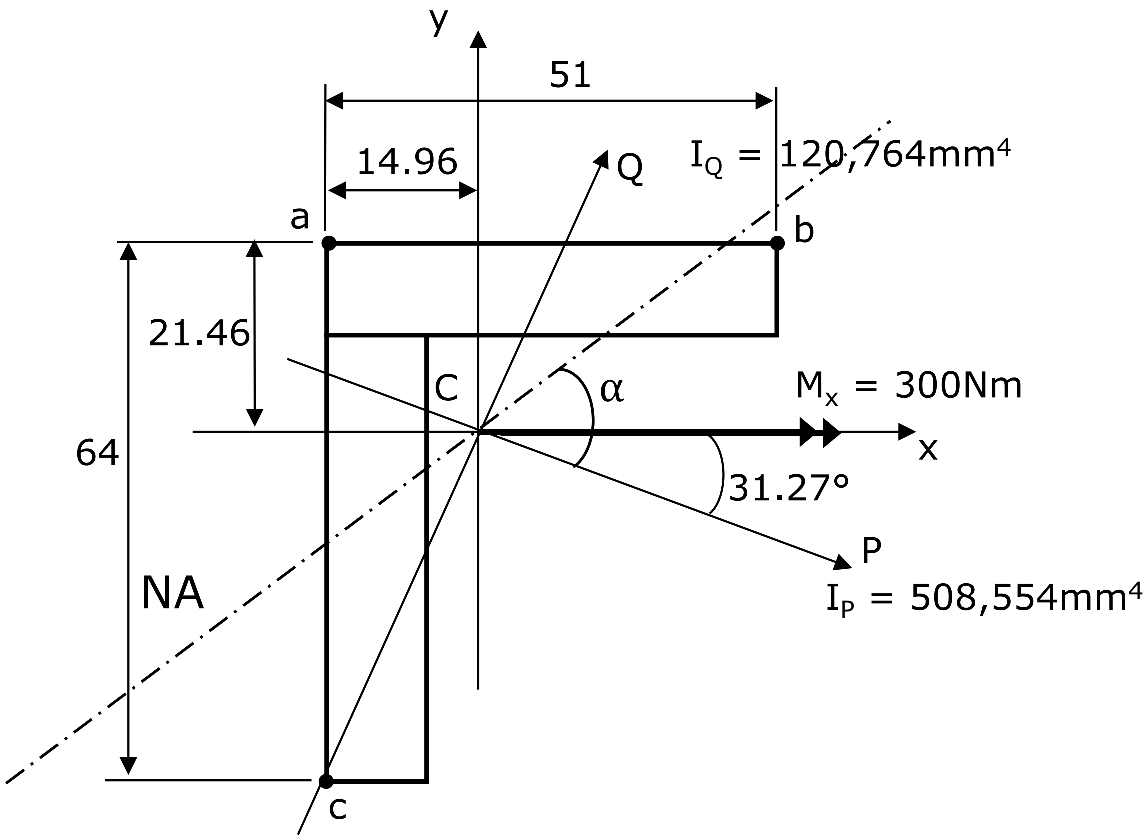
$= -14.96\cos(31.27) + 42.54\sin(31.27)$

$= 9.3$

$Q = x\sin\theta + y\cos\theta$

$= -14.96\sin(31.27) - 42.54\cos(31.27)$

$= -44.12$



$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{256{,}419.2 \times -44.12}{508{,}554} - \frac{155{,}721.5 \times 9.3}{120{,}764}$$

$$\therefore \boldsymbol{\sigma_{b_c} = -34.24\ \text{MPa (compressive)}}$$