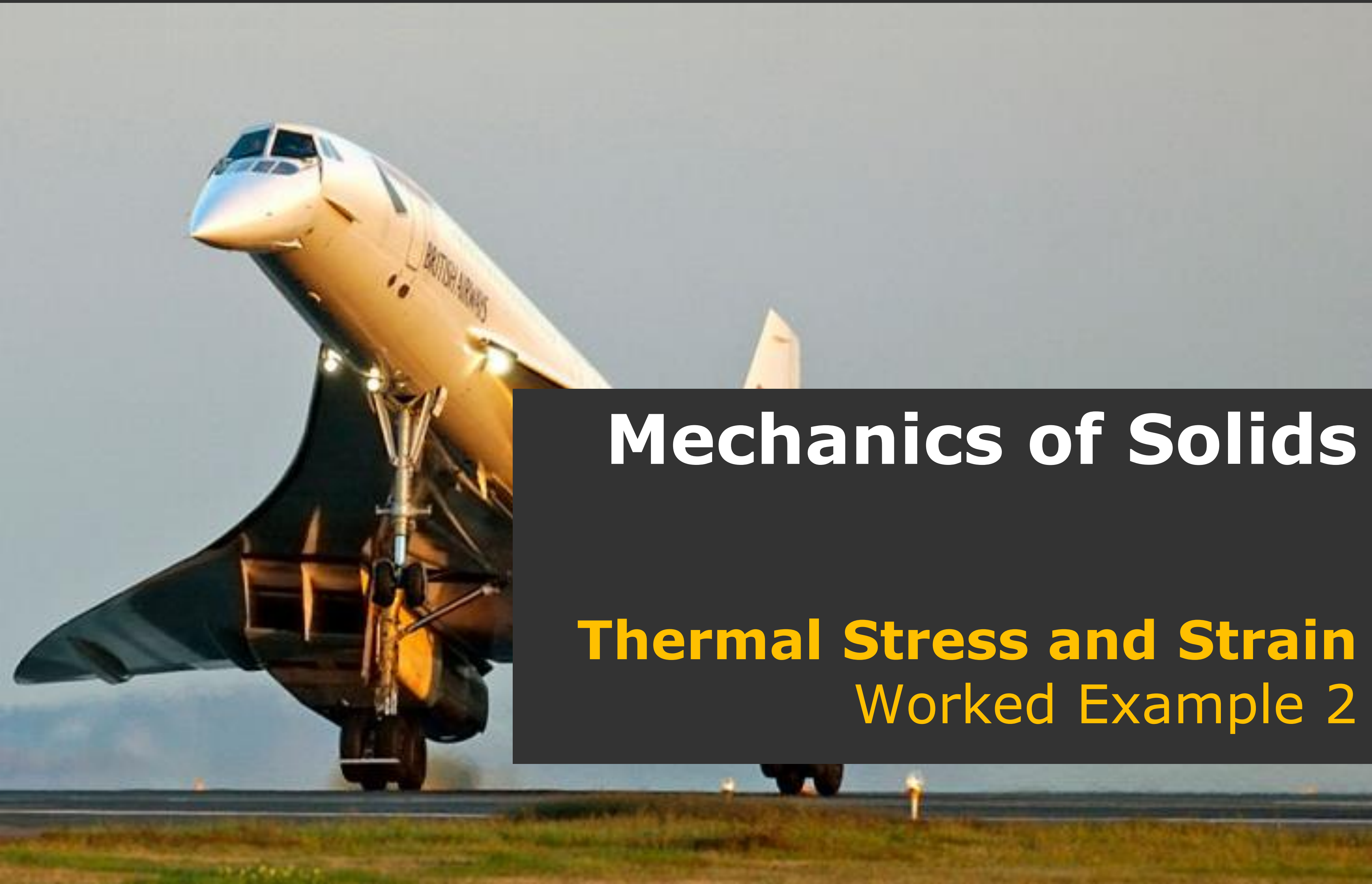


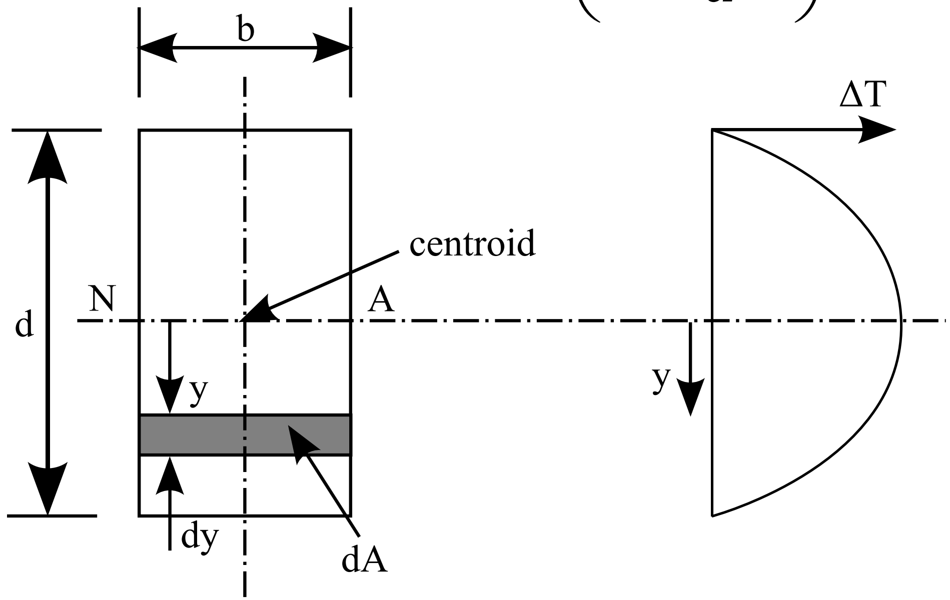
Mechanics of Solids

Thermal Stress and Strain
Worked Example 2

Example 2

A rectangular beam, of width *b* and depth *d*, has a temperature variation given by:

$$\Delta T(y) = \Delta T_{max}\left(1 - \frac{4y^2}{d^2}\right)$$



There is no restraint or applied loading, so $P = M = 0$. Obtain the stress distribution

Example 2

Axial Force Equilibrium

Equation (5) $$P = E\bar{\varepsilon}A - E\alpha\int_A \Delta T dA$$

So, $$0 = E\bar{\varepsilon}bd - E\alpha\int_{-\frac{d}{2}}^{\frac{d}{2}} \Delta T_{max}\left(1 - \frac{4y^2}{d^2}\right)bdy$$

Example 2

Axial Force Equilibrium

Rearranging, $$\bar{\varepsilon} = \frac{\alpha}{d}\Delta T_{max}\int_{-\frac{d}{2}}^{\frac{d}{2}}\left(1 - \frac{4y^2}{d^2}\right)dy$$

$$\bar{\varepsilon} = \frac{\alpha}{d}\Delta T_{max}\left[y - \frac{4y^3}{3d^2}\right]_{-\frac{d}{2}}^{\frac{d}{2}}$$

$$\bar{\varepsilon} = \frac{2}{3}\alpha\Delta T_{max}$$

Example 2

Moment Equilibrium

Equation (7) could be used to determine 1/R using M = 0

$$M = \frac{EI}{R} - E\alpha \int_A \Delta T y dA$$

But from symmetry, we can see that 1/R = 0 (radius is infinite)

Example 2

Stress Distribution

Equation (3) $$\sigma_x = E\left(\bar{\varepsilon} + \frac{y}{R} - \alpha\Delta T\right)$$

Substituting in for $\bar{\varepsilon}$, 1/R and the given temperature variation gives

$$\sigma_x = E\left(\frac{2}{3}\alpha\Delta T_{max} + 0 - \alpha\Delta T_{max}\left(1 - \frac{4y^2}{d^2}\right)\right)$$

$$\sigma_x = E\alpha\Delta T_{max}\left(\frac{4y^2}{d^2} - \frac{1}{3}\right)$$

Example 2

Evaluate Stress Distribution

At y = 0

$$\sigma_x = \frac{-E\alpha\Delta T_{max}}{3}$$

At y = ±d/2

$$\sigma_x = \frac{2E\alpha\Delta T_{max}}{3}$$

Example 2

Evaluate Stress Distribution

σ_x = 0 when $$\frac{4y^2}{d^2} = \frac{1}{3}$$ (from $\sigma_x = E\alpha\Delta T_{max}\left(\frac{4y^2}{d^2} - \frac{1}{3}\right)$)

i.e. $$y = \sqrt{\frac{1}{12}d^2}$$

or $y = \pm 0.287d$

Example 2

Evaluate Stress Distribution

Taking the points we know and plotting a transition between them gives: