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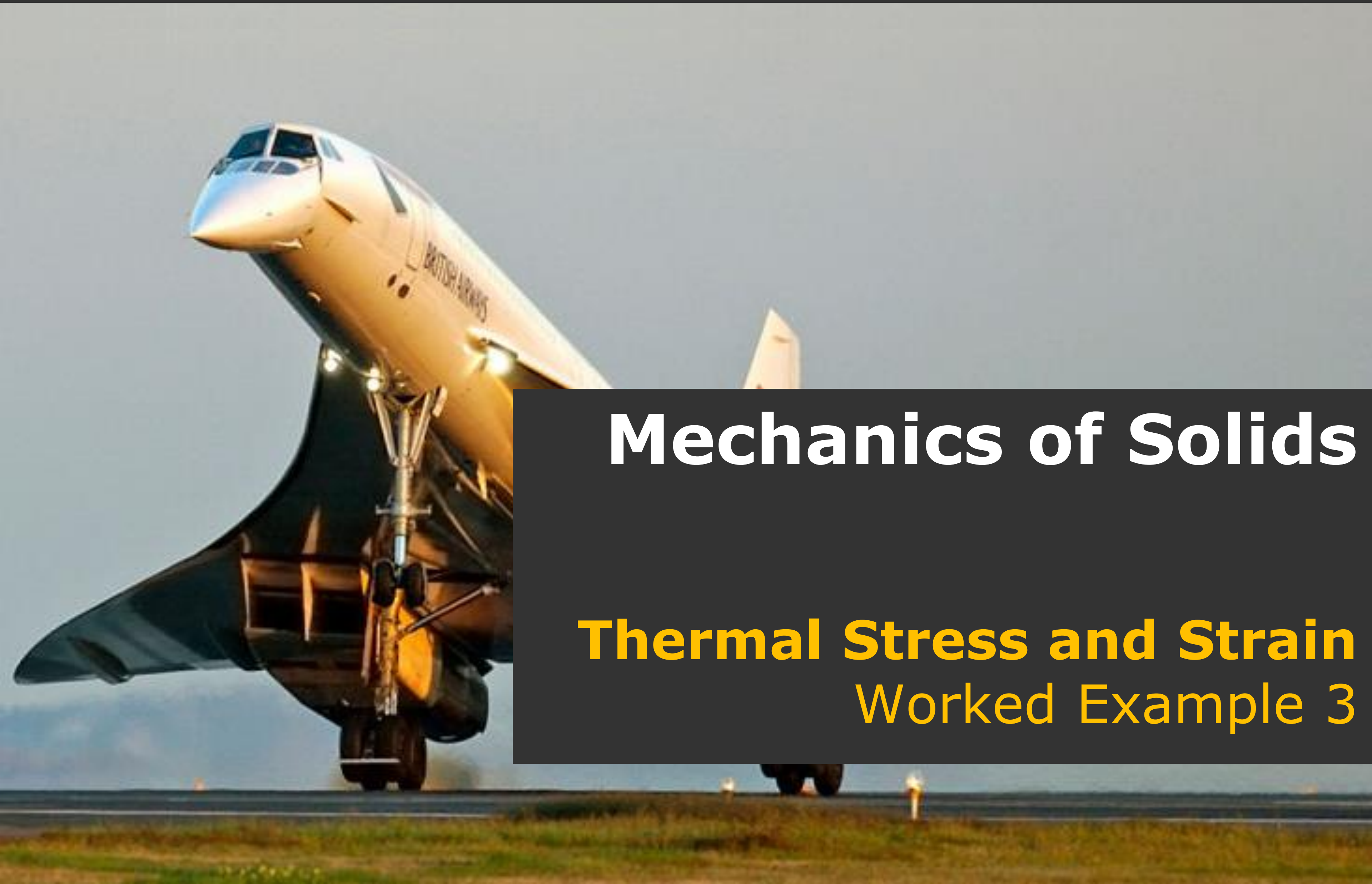


Mechanics of Solids

Thermal Stress and Strain

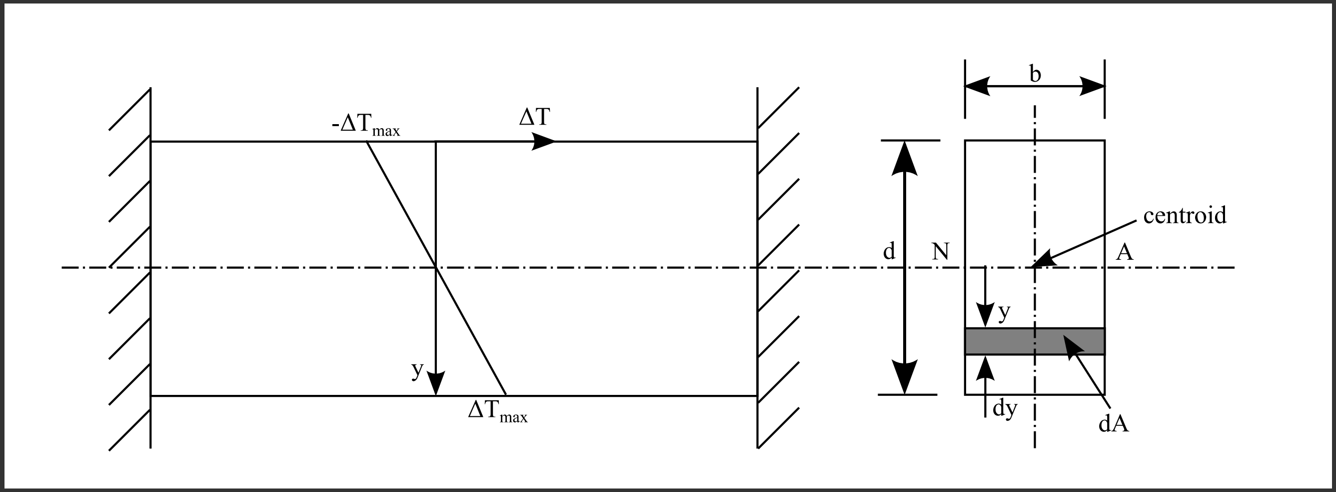
Worked Example 3

Example 3

A rectangular beam, of width b and depth d, has a temperature variation given by:

$$\Delta T(y) = \Delta T_{max} \frac{2y}{d}$$

and is constrained so that $\bar{\varepsilon} = 0$ and $1/R = 0$



Determine the reaction forces and the stress distribution

Example 3

Axial Force Equilibrium

Equation (5) $$P = E\bar{\varepsilon}A - E\alpha\int_A \Delta T dA$$

So, $$P = E\bar{\varepsilon}bd - E\alpha\int_{-\frac{d}{2}}^{\frac{d}{2}} \Delta T_{max}\left(\frac{2y}{d}\right) bdy$$

Example 3

Axial Force Equilibrium

Evaluate the integral,

$$\int_{-\frac{d}{2}}^{\frac{d}{2}} \Delta T_{max} \left(\frac{2y}{d}\right) b dy = \frac{2\Delta T_{max} b}{d} \int_{-\frac{d}{2}}^{\frac{d}{2}} y dy = \frac{2\Delta T_{max} b}{d} \left[\frac{y^2}{2}\right]_{-\frac{d}{2}}^{\frac{d}{2}} = 0$$

And given that $\bar{\varepsilon} = 0$ gives $P = 0$

Example 3

Moment Equilibrium

Recalling Equation (7):

$$M = \frac{EI}{R} - E\alpha \int_A \Delta T y dA$$

$$\int_A \Delta T y dA = \frac{2\Delta T_{max} b}{d} \int_{-\frac{d}{2}}^{\frac{d}{2}} y^2 dy = \frac{2\Delta T_{max} b}{d} \left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{\frac{d}{2}}$$

$$= \frac{2\Delta T_{max} b}{d} \left[\frac{d^3}{24} - \frac{-d^3}{24}\right] = \frac{\Delta T_{max} b d^2}{6}$$

Example 3

Moment Equilibrium

As 1/R = 0, substituting into Equation (7) gives

$$M = \frac{EI}{R} - E\alpha \int_A \Delta T y dA$$

$$M = \frac{-E\alpha\Delta T_{max} b d^2}{6}$$

Example 3

Stress Distribution

Equation (3)

$$\sigma_x = E\left(\bar{\varepsilon} + \frac{y}{R} - \alpha\Delta T\right)$$

Substituting in for $\bar{\varepsilon} = 0$ and 1/R = 0 gives:

$$\sigma_x = -E\alpha\Delta T$$

Substituting in for the temperature variation:

$$\sigma_x = -E\alpha\Delta T_{max}\frac{2y}{d}$$

Example 3

Evaluate Stress Distribution

At y=+d/2 $$\sigma_x = -E\alpha\Delta T_{max}$$

At y=-d/2 $$\sigma_x = E\alpha\Delta T_{max}$$