University of Nottingham

Department of Mechanical, Materials and Manufacturing Engineering

Thermofluids 3 (MMME3081)

Formulae Summary Sheet

(Note that some symbols have different meanings in different equations, (eg C could be velocity or Capacity Ratio)

Relationship between mass and moles

$$\frac{m}{\tilde{m}} = n \quad \text{for any substance}$$

$$\frac{V_i}{V} = \frac{p_i}{p} = \frac{n_i}{n} \quad \text{for a perfect gas}$$

Equation of state for a perfect gas

$$pV = n\tilde{R}\,T \quad \text{or} \quad pV = mRT \qquad pv = RT$$

$$\tilde{R} = 8314 \text{ J/kmol K} \qquad R = \tilde{R}/\tilde{m}$$

Internal energy, specific heat capacity and enthalpy

$$\Delta U = mc_v\Delta T \qquad \Delta H = mc_p\Delta T \quad \text{(when there is no change of phase)}$$

$$\Delta u = c_v\Delta T \qquad \Delta h = c_p\Delta T \qquad H = U + pV$$

First Law of Thermodynamics

Non flow energy equation (NFEE)

For a cycle: $Q_{net} + W_{net} = 0$

For a process $Q + W = U_2 - U_1$

Steady flow energy equation (SFEE) for a single mass flow passing through a control volume

$$\dot{Q} + \dot{W} = \dot{m}(h_2 - h_1) + \frac{\dot{m}}{2}(C_2^2 - C_1^2) + \dot{m}g(z_2 - z_1)$$

$$q + w = (h_2 - h_1) + \frac{(C_2^2 - C_1^2)}{2} + g(z_2 - z_1)$$

Non-Steady Flow Energy Equation

$$\dot{Q} + \dot{W} + p\frac{dV}{dt} = \sum \dot{m}_o\left(h_o + \frac{c_o^2}{2} + gz_o\right) - \sum \dot{m}_i\left(h_i + \frac{c_i^2}{2} + gz_i\right) + \frac{d}{dt}m_{cv}\left(u_{cv} + \frac{c_{cv}^2}{2} + gz_{cv}\right)$$

For continuity of mass $\dot{m}_1 = \dot{m}_2$

For a 1-dimensional flow $\dot{m} = \rho AC$

Second Law of Thermodynamics

For an isolated system

$$\Delta S_{TOTAL} = \Delta S_{system} + \Delta S_{surroundings} \geq 0$$

Definition of entropy

$$S_2 - S_1 = \int_1^2 \left(\frac{dQ}{T}\right)_{reversible}$$

Thermal (1st Law) Efficiency of a Heat Engine

$$\eta = \frac{net\ work\ done}{heat\ supplied} = \left|\frac{W}{Q_{hot}}\right|$$

Maximum Efficiency of a heat engine (Carnot Efficiency) (reversible engine)

$$\eta_{rev} = 1 - \frac{T_{COLD}}{T_{HOT}}$$

Isentropic efficiency η_{isen}

$$\eta_t = \frac{W_{out}}{W_{isentropic}} \quad \text{for a turbine} \quad \left[= \frac{(h_1 - h_2)}{(h_1 - h_2)_{isentropic}} \quad \text{adiabatic turbine} \right]$$

$$\eta_c = \frac{W_{isentropic}}{W_{in}} \quad \text{for a compressor or pump} \quad \left[= \frac{(h_2 - h_1)_{isentropic}}{(h_2 - h_1)} \quad \text{adiabatic compressor} \right]$$

Relationships for Perfect Gases

$$c_p - c_v = R \qquad c_p/c_v = \gamma$$

$$s_2 - s_1 = c_v \log_e\left(\frac{T_2}{T_1}\right) + R\log_e\left(\frac{V_2}{V_1}\right)$$

$$s_2 - s_1 = c_p \log_e\left(\frac{T_2}{T_1}\right) - R\log_e\left(\frac{p_2}{p_1}\right)$$

Isothermal Process $p_1V_1 = p_2V_2 = constant$

Reversible adiabatic (S = constant, Q = 0)

$$p_1V_1^\gamma = p_2V_2^\gamma = constant \qquad (\gamma = c_p/c_v)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Polytropic process $p_1V_1^n = p_2V_2^n = constant$ (n = polytropic index)

<u>Work Transfer for Reversible Processes</u> (ie no friction)

NON-FLOW PROCESSES

$W = -\int_1^2 p\,dV$	general case
$W = p\left(V_2 - V_1\right)$	constant pressure
$W = 0$	constant volume
$W = mRT\ln\left(\frac{V_2}{V_1}\right)$	isothermal (perfect gas)
$W = \frac{\left(p_2V_2 - p_1V_1\right)}{1-n}$	polytropic pV^n = constant

FLOW PROCESSES (neglecting changes in potential and kinetic energy)

$\dot{W} = \dot{m}\int_1^2 v\,dp$	general case
$W = 0$	constant pressure
$\dot{W} = \dot{m}v\left(p_2 - p_1\right)$	constant specific volume (density)
$\dot{W} = \dot{m}RT\ln\left(\frac{p_2}{p_1}\right)$	isothermal (perfect gas)
$\dot{W} = \dot{m}\frac{n}{\left(n-1\right)}\left(p_2v_2 - p_1v_1\right)$	polytropic pV^n = constant

<u>Combustion</u>

A typical fuel combustion reaction equation (complete combustion) is:

$$\underbrace{\underbrace{CH_4}_{\text{Fuel}} + \underbrace{xO_2 + yN_2}_{\text{Air}}}_{\text{Combustion reactants}} \rightarrow \underbrace{aCO_2 + bH_2O + cO_2 + dN_2}_{\text{Combustion products}}$$

Approximate atomic/molecular weight

	$\tilde{m}$ (kg/kmol)		$\tilde{m}$ (kg/kmol)
H	1	H_2O	18
C	12	CO_2	44
O	16	CO	28
N	14	CH_4 (methane)	16
S	32	C_3H_8 (propane)	44
Cl	35.5	C_4H_{10} (butane)	58
H_2	2	SO_2	64
O_2	32	HCl	36.5
N_2	28		
Air	29		

Excess Air = (Actual air for combustion – Stoichiometric air for combustion)/Stoichiometric air

Gross and Net Calorific Values

Calorific values may be expressed in either of two ways:

If the CV is evaluated with the water vapour in the products of combustion as a **liquid** then it is known as *Gross Calorific Value* (GCV) (or *Higher Calorific Value* (HCV) or *Higher Heating Value (HHV)*).

If the CV is evaluated with the water vapour in the products of combustion as a **vapour** then it is known as *Net Calorific Value* (NCV) (or *Lower Calorific Value* (LCV) or *Lower Heating Value (LHV)*).

$$GCV = NCV + m_i\, h_{fg} \text{ (at 25°C)}$$

h_{fg} at 25°C = 2442 kJ/kg

Steady Flow Energy Equation - applied to combustion

$$\dot{Q} + \dot{W} = \sum_{Products} \dot{m}_i \left(h_i - h_{i0}\right) - \sum_{Reactants} \dot{m}_i \left(h_i - h_{i0}\right) + \dot{m}_f\, \Delta h_0$$

COMBUSTION EQUILIBRIUM

See pages 18, 19, 20 and 21 in Rogers and Mayhew Tables.

Equilibrium constant

$$K^{\Theta} = \frac{\left(p_C\right)^c\left(p_D\right)^d}{\left(p_A\right)^a\left(p_B\right)^b}\left(p^{\Theta}\right)^{(a+b-c-d)}$$

$$K^{\Theta} = \frac{\left(x_C\right)^c\left(x_D\right)^d}{\left(x_A\right)^a\left(x_B\right)^b}\left(\frac{p_{total}}{p^{\Theta}}\right)^{(c+d-a-b)}$$

$$K^{\Theta} = e^{-\left(\frac{\Delta\tilde{g}^{\Theta}_{(T)}}{\tilde{R}T}\right)}$$

Use of Enthalpy of Formation

$$\tilde{h}_T = \tilde{h}_{f0} + \Delta\tilde{h}_{T-0}$$

Heat Transfer

Conduction

Steady conduction through a plane wall or shape S

$$\dot{q} = \frac{kA}{\Delta x}(T_1 - T_2), \quad \dot{q} = kS(T_1 - T_2)$$

$$R_{th} = \frac{\Delta x}{kA} \qquad \dot{q} = \frac{(T_1 - T_2)}{R_{th}}$$

For a cylindrical layer:

$$R_{th} = \frac{\log_e\left(\frac{r_o}{r_i}\right)}{2\pi kL},$$

Fin information for a long fin: $m = \sqrt{\frac{hP}{kA}}, \quad \theta = \theta_0 e^{-mx}$

Fin efficiency for an insulated tip fin: $\eta = \frac{\tanh(m\,L_c)}{m\,L_c}$

Convection Newton's Law of Cooling:

$\dot{q} = hA(T_s - T_f)$ *(for heat transfer from a surface to a fluid)*

Nusselt number (Ratio of convection heat transfer to conduction in a fluid)

$Nu_d = \frac{hd}{k}$ (based on diameter) or $Nu_L = \frac{hL}{k}$ based on length

Reynolds number (Ratio of dynamic to viscous forces in a fluid)

$\mathrm{Re}_d = \frac{\rho u d}{\mu}$ or $\mathrm{Re}_L = \frac{\rho u L}{\mu}$

Prandtl number (Ratio of momentum diffusivity (viscosity) to thermal diffusivity)

$$\Pr = \frac{\mu c_p}{k}$$

Grashof number (Ratio of buoyancy to viscous forces in a fluid)

$$Gr = \frac{g\beta L^3 \rho^2 \Delta T}{\mu^2}$$

(Note: ß = 1/T for perfect gases)

Rayleigh Number Ra = Gr.Pr, **Stanton Number** St = Nu/Re.Pr

Biot Number Bi = hV/kA

Fourier number Fo = $\alpha t/L^2$

Radiation

Stefan-Boltzmann Law.

$$\dot{q}_b = A\sigma T^4$$

where σ = Stefan-Boltzmann constant = 5.67×10^{-8} W/m²K⁴

Radiation exchange (black surfaces)

$$\dot{q}_{b12} = A_1 F_{1-2}\sigma(T_1^4 - T_2^4)$$

Radiation exchange (two grey surfaces)

$$\dot{q}_{12} = A_1 \Im_{1-2}\sigma(T_1^4 - T_2^4)$$

where $\Im_{1-2}$ is the *grey body exchange factor*

$$\Im_{1-2} = \frac{1}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{1-2}} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

An equivalent heat transfer coefficient can be defined for solving radiation problems.

$h_{rad} = \Im_{1-2}\sigma(T_1 + T_2)(T_1^2 + T_2^2)$

Heat Exchangers

Capacity Ratio C

If $\dot{m}_h c_{ph} > \dot{m}_c c_{pc}$ then $C = \frac{\dot{m}_c\, c_{pc}}{\dot{m}_h\, c_{ph}}$

If $\dot{m}_h c_{ph} < \dot{m}_c c_{pc}$ then $C = \frac{\dot{m}_h\, c_{ph}}{\dot{m}_c\, c_{pc}}$

Effectiveness ε

$$\varepsilon = \frac{\text{Actual heat transferred}}{\text{Maximum possible heat transfer}}$$

$$\varepsilon = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})} \quad \text{if } \dot{m}_h c_{ph} < \dot{m}_c c_{pc}$$

$$\varepsilon = \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})} \quad \text{if } \dot{m}_h c_{ph} > \dot{m}_c c_{pc}$$

An alternative way of expressing effectiveness is:

$$\varepsilon = \frac{\Delta T\ (\text{fluid with minimum capacity})}{\text{Maximum temperature difference in heat exchanger}}$$

Heat Transfer in heat exchangers

$$\dot{q} = UA\,\Delta T_m = \dot{m}_h c_{ph} \Delta T_h = \dot{m}_c c_{pc} \Delta T_c$$

ΔT_m is logarithmic mean temperature difference (LMTD)

$$LMTD = \frac{(\theta_i - \theta_o)}{\log_e\left(\frac{\theta_i}{\theta_o}\right)}$$

where θ_i = temperature difference between steams at hot inlet, and θ_o = temperature difference between streams at hot outlet.

ε-NTU Method

$$\frac{UA}{(\dot{m}c_p)_{min}} = \text{Number of transfer units } (NTU)$$

for a parallel-flow heat exchanger $\varepsilon = \dfrac{1 - e^{-NTU(1+C)}}{(1+C)}$

for a counter-flow heat exchanger $\varepsilon = \dfrac{1 - e^{-NTU(1-C)}}{1 - Ce^{-NTU(1-C)}}$

for a balanced (C=1) counter-flow heat exchanger $\varepsilon = \dfrac{NTU}{1+NTU}$

Thermal Power Cycles

GENERAL

Thermal Efficiency $\eta = \dfrac{\text{net work output}}{\text{external heat supplied}}$

Work Ratio $r_w = \dfrac{\text{net work output}}{\text{gross work output}}$

STEAM PLANT

Specific steam consumption $SSC = \dfrac{\text{mass flow of steam}}{\text{net power output}}$

GAS TURBINE PLANT

Pressure ratio $r_p = \dfrac{p_2}{p_1}$ *pressure ratio across compressor*

Specific Work *net work output per kg of mass flow*

Maximum thermal efficiency for ideal cycle $\eta = 1 - \left(\dfrac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$

Work ratio for an ideal cycle $r_w = 1 - \left(\dfrac{T_1}{T_3}\right)(r_p)^{\frac{\gamma-1}{\gamma}}$

Polytropic efficiency of **compressor** $\eta_{\infty c}$

Polytropic efficnency of **turbine** $\eta_{\infty t}$

Temperature change in compressor (using polytropic efficiency) $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_{\infty c}}}$

Temperature change in turbine (using polytropic efficiency) $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{(\gamma-1)\eta_{\infty t}}{\gamma}}$

Effectiveness of recuperator in a gas turbine is defined as: $\varepsilon = \dfrac{T_{2a} - T_2}{T_4 - T_2}$

Where T_2 is compressor outlet temp, T_4 is turbine outlet temp and T_{2a} is air outlet temp from recuperator.

Exergy

Thermal Exergy – due to heat transfer

$$\dot{E}^Q = \dot{Q}_1\left(\frac{(T_1 - T_0)}{T_1}\right)$$

T_0 is the environmental temperature

Non-Flow Exergy

$$\Xi = U_1 - U_0 + p_0(V_1 - V_0) - T_0(S_1 - S_0)$$

Flow Exergy

$$\dot{E} = \dot{m}\left[(h_1 - h_0) - T_0(s_1 - s_0)\right]$$

neglecting the potential and kinetic energy terms.

Exergy Balance

$$\text{Exergy in} = \text{Exergy out} + \text{Irreversibility}$$

Rational Efficiency

$$\psi = \frac{\sum \Delta\dot{E}_{output}}{\sum \Delta\dot{E}_{input}} = \frac{\text{change in exergy of output stream}}{\text{change in exergy of input stream}} = \frac{\text{useful exergy output}}{\text{exergy input}}$$

Turbomachinery

Formulae which may be used for turbine design parameters and general turbomachinery equations:

$$\psi = \frac{h_0}{U^2} = \frac{w_{\theta 3} - w_{\theta 2}}{U} = \emptyset(tan\beta_3 + tan\beta_2)$$

$$R = \phi\,(tan\beta_3 - tan\beta_2)/2$$

$$\phi = \frac{c_x}{U_m}$$