University of Nottingham
Department of Mechanical, Materials and Manufacturing Engineering

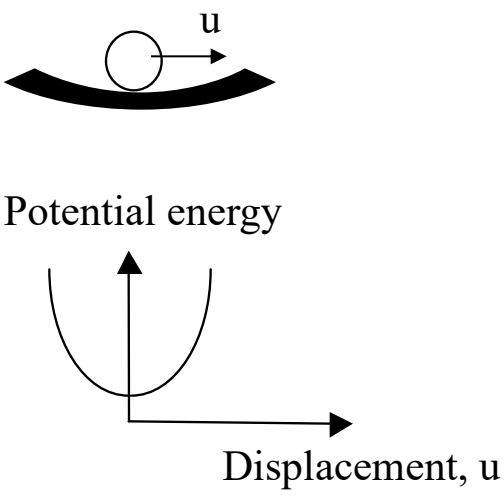
Computer Modelling Techniques

FE-01-03

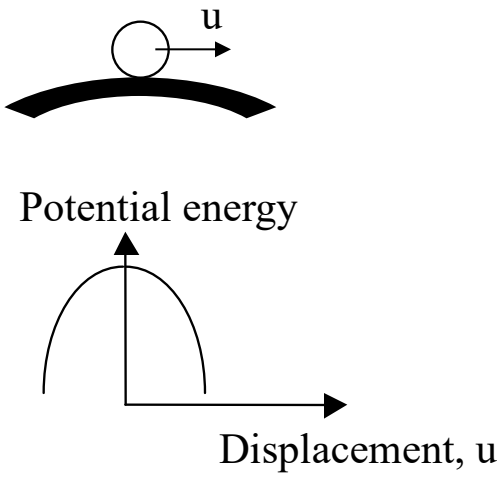
ENERGY APPROACHES

1.3 Energy Approaches

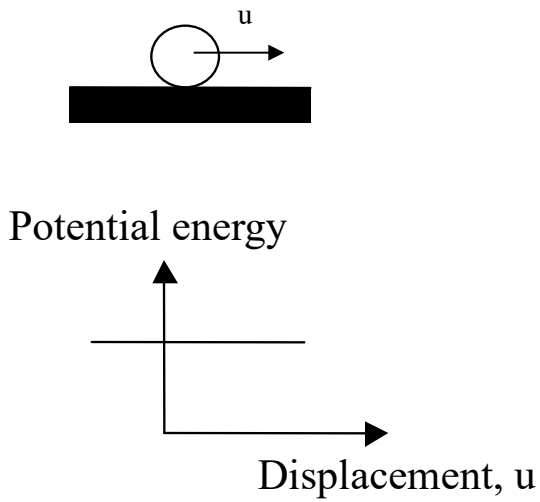
1.3.1 Stable and Unstable Problems



Stable equilibrium



Unstable equilibrium



Neutral equilibrium

1.3.2 Strain Energy

This strain energy is released upon the removal of the applied loads and the body returns to its un-deformed state.

$$U = \frac{1}{2}[\sigma][\varepsilon] \times volume$$

If the material behaviour is non-linear, a more general expression can be written as follows:

$$U = \int_{v} \int_{\varepsilon} \sigma \, d\varepsilon \, dV$$

1.3.3 Work Done by External Forces

Another form of potential energy arises from the work done by the external forces that cause deformation of the body. This energy can be written as follows:

$$W = \sum_i F_i u_i$$

where i is any point where the force F_i causes a displacement u_i.

1.3.4 The Principle of Minimum Total Potential Energy (T.P.E)

- The total potential energy (*T.P.E.*) can be expressed as the difference between the strain energy and the work done by the external forces, as follows:

$$T.P.E. = U - W$$

- The ***principle of minimum total potential energy*** states that when the body is in equilibrium, the value of the *T.P.E.* must be 'stationary' with respect to the variables of the problem.

- The equilibrium is **stable if the *T.P.E.* is minimum**

Important

- In most FE formulations, **the displacement, *u*, is chosen as the unknown variables** of the problem, i.e.

$$\frac{\partial(T.P.E.)}{\partial u} = 0$$